**Dawson College**: **Calculus I** (SCIENCE): 201-SN2-RE-S14: Fall 2024: **Quiz 6** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

**Question 1.** Consider the function and its derivatives

$$f(x) = \frac{e^x}{x^2},\ f'(x) = \frac{xe^x - 2e^x}{x^3},\ f''(x) = \frac{x^2e^x - 4xe^x + 6e^x}{x^4}$$

a. *(4 marks)* Find the domain, intercepts and asymptotes of $f(x)$ (if they exist).

Domain of $f(x)$: $\mathbb{R}\setminus\{0\}$

Intercepts: no y-int since $f(x)$ is not defined at $x=0$

no x-int since $e^x \neq 0$

Asymptotes: $\lim_{x\to 0^\pm} f(x) = \infty$ $\therefore$ vertical asymptote at $x=0$.

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{e^x}{x^2} \overset{\hat{H}}{=} \lim_{x\to\infty} \frac{e^x}{2x} \overset{\hat{H}}{=} \lim_{x\to\infty} \frac{e^x}{2} = \infty$$

$$\lim_{x\to-\infty} f(x) = \text{''} = \text{''} = \lim_{x\to-\infty} \frac{e^x}{2} = 0$$ $\therefore$ horizontal asymptotes $y=0$.

b. *(4 marks)* Find the intervals where $f(x)$ is increasing/decreasing and the points where local maxima and minima occur (if they exist).

$0 = f'(x)$

$0 = \frac{xe^x - 2e^x}{x^3}$

$0 = e^x(x-2)$

$0 = x-2$

$x = 2$

$\therefore$ critical point at $x=2$

| | $(-\infty, 0)$ | $(0, 2)$ | $(2, \infty)$ |
|---|---|---|---|
| test point | $-1$ | $1$ | $3$ |
| $f'(t.p.)$ | $f'(-1) > 0$ | $f'(1) < 0$ | $f'(3) > 0$ |
| inc./dec. | $\nearrow$ | $\searrow$ | $\nearrow$ |

$\therefore$ local min. at $x=2$

$\therefore$ point of local min. is $(2, f(2)) = \left(2, \frac{e^2}{2^2}\right) \doteq (2, 1.8)$

c. *(4 marks)* Find the intervals where $f(x)$ is concave upward/downward and the points of inflection (if they exist).

$0 = f''(x)$

$0 = \frac{x^2e^x - 4xe^x + 6e^x}{x^3}$

$0 = e^x(x^2 - 4x + 6)$

$0 = x^2 - 4x + 6$

irreducible since

$\Delta = (-4)^2 - 4(1)(6)$

$= 16 - 24$

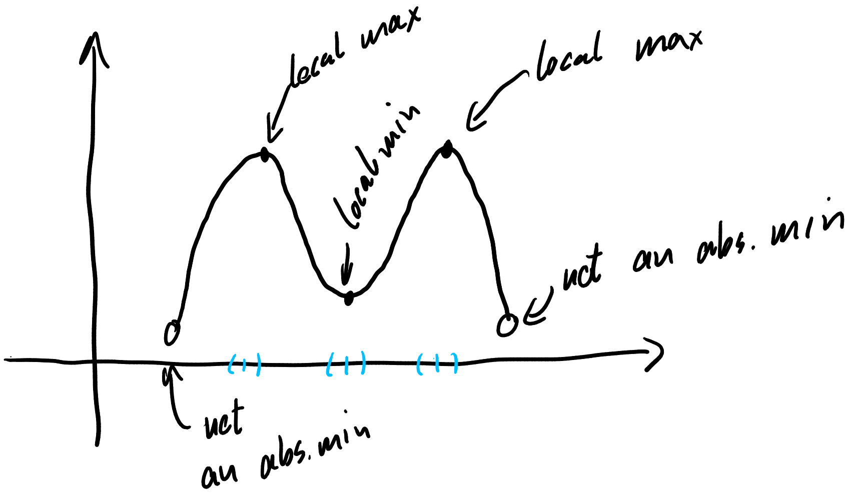
$= -8 < 0$

| | $(-\infty, 0)$ | $(0, \infty)$ |
|---|---|---|
| test point | $-1$ | $1$ |
| $f''(t.p.)$ | $f''(-1) > 0$ | $f''(1) > 0$ |
| Concavity | $\cup$ | $\cup$ |

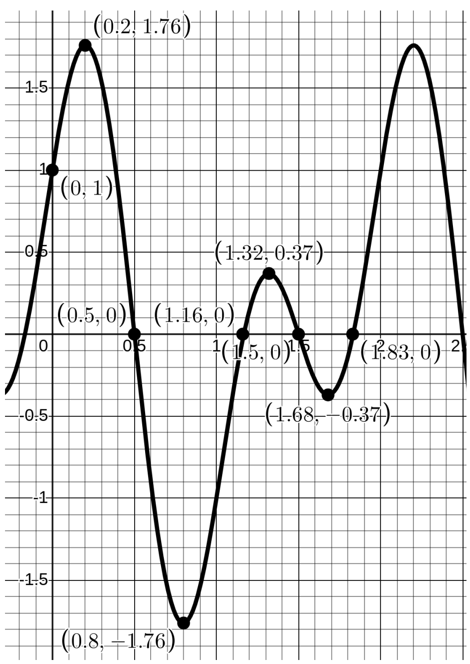
no inflection point.

d. *(4 marks)* Sketch the graph of $f(x)$. Show clearly all important points.

**Question 2.** *(3 marks)* Sketch the graph of a function that has two local maxima, one local minimum, and no absolute minimum.




**Question 3.** *(4 marks)* The graph of $f'(x)$ is given below, on the interval $[0,2]$ find the intervals of concavity and the x-coordinates of the inflection points of $f(x)$ if any. Justify.




The slope of the tangents of $f'(x)$ is positive on: $[0, 0.2)$, $(0.8, 1.32)$, $(1.68, 2]$ *

and negative on $(0.2, 0.8)$, $(1.32, 1.68)$ *

So $f''(x) > 0$ on * and $f''(x) < 0$ on *

$\therefore$ Inflection points at $x = 0.2, 0.8, 1.32, 1.68$.

**Question 4.** *(4 marks)* Find the absolute maximum and absolute minimum values of $f(x) = 3x^4 - 4x^3 - 12x^2 + 1$ on the interval $[-2, 3]$.

Let's find the critical points of $f(x)$

$f'(x) = 12x^3 - 12x^2 - 24x$

$0 = f'(x)$

$0 = 12x^3 - 12x^2 - 24x$

$0 = 12x(x^2 - x - 2)$

$0 = 12x(x-2)(x+1)$

$x = 0 \quad x = 2 \quad x = -1$

$f(-2) = 3(-2)^4 - 4(-2)^3 - 12(-2)^2 + 1 = 33$

$f(-1) = 3(-1)^4 - 4(-1)^3 - 12(-1)^2 + 1 = -4$

$f(0) = 1$

$f(2) = 3(2)^4 - 4(2)^3 - 12(2)^2 + 1 = -31$

$f(3) = 3(3)^4 - 4(3)^3 - 12(3)^2 + 1 = 28$

$\therefore$ abs. max of 33 at $x = -2$

abs. min of $-31$ at $x = 2$.

**Dawson College**: **Calculus I** (Science): 201-SN2-RE-S14: Fall 2024: **Quiz 6** name: ____________