Dawson College: **Calculus I** (SCIENCE): 201-SN2-RE-S14: Fall 2024: **Quiz 7** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(5 marks)* Apply the Mean Value Theorem for $f(x) = 4x + \frac{1}{4x}$ over the interval $[1/4, 2]$.

$f(x)$ is continuous on $[1/4, 2]$

$f(x)$ is differentiable on $(1/4, 2)$

$\therefore$ by the MVT $\exists c$ s.t. $f'(c) = \dfrac{f(b)-f(a)}{b-a}$. Lets find the 'c's.

$f'(x) = 4 - \dfrac{1}{4x^2}$

$$4 - \frac{1}{4c^2} = \frac{f(2) - f(1/4)}{2 - 1/4}$$

$$4 - \frac{1}{4c^2} = \frac{4(2) + \frac{1}{4(2)} - \left[4(\frac{1}{4}) + \frac{1}{4(\frac{1}{4})}\right]}{\frac{7}{4}}$$

$$4 - \frac{1}{4c^2} = \frac{8 + \frac{1}{8} - [1 + 1]}{\frac{7}{4}}$$

$$4 - \frac{1}{4c^2} = \frac{49}{8} / \frac{7}{4}$$

$$4 - \frac{1}{4c^2} = \frac{49}{8} \cdot \frac{4}{7}$$

$$4 - \frac{1}{4c^2} = \frac{7}{2}$$

$$4 - \frac{7}{2} = \frac{1}{4c^2}$$

$$\frac{1}{2} = \frac{1}{4c^2}$$

$$c^2 = \frac{1}{2}$$

$$c = \pm \frac{1}{\sqrt{2}}$$

$\rightarrow -\frac{1}{\sqrt{2}} \notin [1/4, 2]$

but $c = \frac{1}{\sqrt{2}} \in [1/4, 2]$

Question 2. *(5 marks)* Let $f(x) = (x-3)^{-2}$. Show that there is no value c in $(1, 4)$ such that $f(4) - f(1) = f'(c)(4-1)$. Why does this not contradict the Mean Value Theorem?

Lets show that there are no 'c' value s.t.

$f'(x) = \dfrac{-2}{(x-3)^3}$

$$f(4) - f(1) = f'(c)(4-1)$$

$$\frac{1}{(4-3)^2} - \frac{1}{(1-3)^2} = \frac{-2}{(c-3)^3}\, 3$$

$$1 - \frac{1}{4} = \frac{-6}{(c-3)^3}$$

$$\frac{3}{4} = \frac{-6}{(c-3)^3}$$

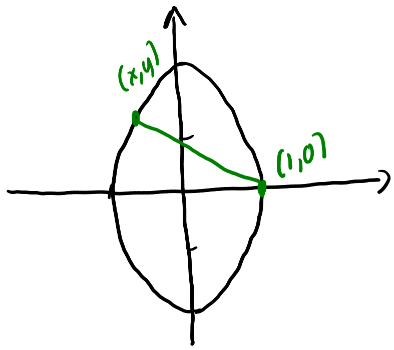
$$(c-3)^3 = -8$$

$$c - 3 = -2$$

$$c = 1 \notin (1,4)$$

This does not contradict the MVT since $f(x)$ is not continuous on $(1,4)$. It has an infinite discontinuity at $x = 3$.

Question 3. *(5 marks)* Find the points on the ellipse $4x^2 + y^2 = 4$ that are the farthest away from the point $(1, 0)$.



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(1-x)^2 + (0 - \pm\sqrt{4-4x^2})^2}$$
$$= \sqrt{(1-x)^2 + (\mp\sqrt{4-4x^2})^2}$$

$$4x^2 + y^2 = 4$$
$$y^2 = 4 - 4x^2$$
$$y = \pm\sqrt{4-4x^2}$$

$$D = d^2 = (1-x)^2 + (\sqrt{4-4x^2})^2$$
$$= 1 - 2x + x^2 + (4 - 4x^2)$$
$$= 5 - 2x - 3x^2 \quad \text{where } x \in [-1, 1]$$

Lets find the critical points of D

$$D' = -2 - 6x$$
$$0 = D'$$
$$0 = -2 - 6x$$
$$2 = -6x$$
$$\frac{-1}{3} = x$$

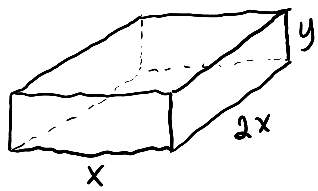
$$D(-1) = 5 - 2(-1) - 3(-1)^2 = 4$$
$$D(1) = 5 - 2(1) - 3(1) = 0$$
$$D(-\tfrac{1}{3}) = 5 - 2(-\tfrac{1}{3}) - 3(-\tfrac{1}{3}) = \frac{20}{3}$$

$\therefore$ at $x = -\frac{1}{3}$ we have an abs max.

$$y = \pm\sqrt{4 - 4(-\tfrac{1}{3})^2}$$
$$= \pm\sqrt{4 - \tfrac{4}{9}} = \pm\sqrt{\tfrac{32}{9}}$$

$\therefore$ farthest points $\left(-\frac{1}{3}, \pm\sqrt{\frac{32}{9}}\right)$.

Question 4. *(5 marks)* Canadian postal regulation requires that the sum of the three dimensions (length+width+height) of a rectangular package to be 3 meters. If the lenght of a package is twice the width, find the dimension of the package of maximum volume that can be mailed.



$$V = x(2x)y = 2x^2y$$

and
$$x + 2x + y = 3$$
$$y = 3 - 3x \qquad x \in (0, 1)$$

$$\therefore V = 2x^2(3 - 3x) = 6x^2 - 6x^3$$

Lets find the critical points

$$V'(x) = 12x - 18x^2$$
$$0 = V'(x)$$
$$0 = 12x - 18x^2$$
$$0 = 6x(2 - 3x)$$

$x = 0 \notin (0,1)$

$$2 - 3x = 0$$
$$3x = 2$$
$$x = \tfrac{2}{3} \in (0,1).$$

	$(0, \frac{2}{3})$	$(\frac{2}{3}, 1)$
test point	$\frac{1}{3}$	$\frac{3}{4}$
$V'(t.p.)$	$V'(\frac{1}{3}) = 2 > 0$	$V'(\frac{3}{4}) < 0$
inc/dec	$\nearrow$	$\searrow$

$\therefore$ abs. max at $x = \frac{2}{3}$

$\therefore$ Length $= 2x = \frac{4}{3}$
width $= \frac{2}{3}$
height $= 3 - 3(\frac{2}{3}) = 1$