Dawson College: **Calculus I** (SCIENCE): 201-SN2-RE-S14: Fall 2024: **Quiz 8** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1.*(5 marks each)* Find the following indefinite integrals

a.

$$\int \frac{2x\sqrt{x}+3x-x^2\sec x\tan x}{x^2}\,dx = \int \frac{2x\sqrt{x}}{x^2} + \frac{3\cancel{x}}{x^{\cancel{2}}} - \frac{\cancel{x^2}\sec x\tan x}{\cancel{x^2}}\,dx$$

$$= \int 2x^{-1/2} + \frac{3}{x} - \sec x\tan x\,dx$$

$$= \frac{2x^{1/2}}{1/2} + 3\ln|x| - \sec x + C$$

$$= 4\sqrt{x} + 3\ln|x| - \sec x + C$$

b.

$$\int \frac{4}{x\sqrt{1-(\ln x)^2}}\,dx = 4\int \frac{1}{\sqrt{1-u^2}}\,du$$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$$= 4\arcsin u + C$$

$$= 4\arcsin(\ln x) + C$$

Question 2. *(5 marks)* Find f where $f''(t) = 2e^t + 3\sin t$, $f(0) = 0$, $f(\pi) = 0$.

$$f'(t) = \int f''(t)\,dt$$
$$= \int 2e^t + 3\sin t\,dt$$
$$= 2e^t - 3\cos t + C$$

$$f(t) = \int f'(t)\,dt$$
$$= \int 2e^t - 3\cos t + c\,dx$$
$$= 2e^t - 3\sin t + Cx + D$$

$$0 = f(0)$$
$$0 = 2e^0 - 3\sin 0 + C(0) + D$$
$$-2 = D$$

$$0 = f(\pi)$$
$$0 = 2e^\pi - 3\sin\pi + C(\pi) - 2$$
$$\frac{2 - 2e^\pi}{\pi} = C$$

$$\therefore f(t) = 2e^t + 3\sin t + \left(\frac{2-2e^\pi}{\pi}\right)t - 2$$

Question 3. *(5 marks)* Find:

$$\int \frac{x^3}{\sqrt{x^2+1}}\,dx = \int \frac{x^2 x}{\sqrt{x^2+1}}\,dx$$

$u = x^2 + 1$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$x^2 = u - 1$

$$= \int \frac{(u-1)}{\sqrt{u}} \frac{du}{2}$$
$$= \frac{1}{2}\int (u-1)u^{-1/2}\,du$$
$$= \frac{1}{2}\int u^{1/2} - u^{-1/2}\,du$$
$$= \frac{1}{2}\left[\frac{u^{3/2}}{3/2} - \frac{u^{1/2}}{1/2}\right] + C$$
$$= \frac{u^{3/2}}{3} - u^{1/2} + C$$
$$= \frac{(x^2+1)^{3/2}}{3} - (x^2+1)^{1/2} + C$$