Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S5: Fall 2024: **Quiz 1** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(3 marks each)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

a. Consider a system of linear equations with augmented matrix A. If there is more than one solution, A has a row of zeros.

False, $\begin{bmatrix} 1 & 1 & 0 \end{bmatrix}$ has ∞-many solutions since $x+y=0$ has a sol. set which is all the points on its graph but the augmented matrix does not have a row of zeros.

b. Multiplying a row of an augmented matrix through by zero is an acceptable elementary row operation.

False, $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ has a unique solution $\begin{cases} x=1 \\ y=1 \end{cases}$ but if $0R_2 \to R_2$ is applied we get $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ and the new system has ∞-many solutions since $x+0y=1$ and $y=t, t\in\mathbb{R} \Rightarrow$ the sol. set is $(x,y)=(1,t), t\in\mathbb{R}$. Elementary row operations must at least preserve the solution set.

Question 2. *(3 marks)* Find (if possible) conditions on a and b such that the system has no solution, one solution, and infinitely many solutions. Justify.

$$\begin{cases} x + ay = 1 \\ 2x + by = 2 \end{cases} \Rightarrow \begin{cases} x = -ay+1 \\ x = -\frac{b}{2}y+1 \end{cases}$$

Note that the two lines have the same intercept $\therefore$ the system is always consistent since they always have the intercept in common.
$\therefore \nexists\, a,b,$ s.t. the system is inconsistent.

If $-a = -\frac{b}{2} \Rightarrow b=2a$ then both lines have the same slope and since they have the same intercept, they are identical. $\therefore$ ∞-many points in common. $\therefore$ ∞-many solutions.

If $b \neq 2a$ then the lines have different slope and a unique point in common, the intercept. $\therefore$ unique solution.

Question 3. *(2 marks)* Consider the following augmented matrix of a consistent linear system.

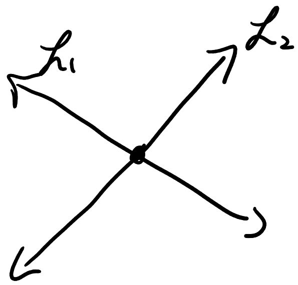
$$\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 2 & 4 & 6 \end{array}\right]$$ is the augmented matrix for the system $\mathcal{L}_1: x+2y=3$, $\mathcal{L}_2: 2x+3y=4$, $\mathcal{L}_3: 2x+4y=6$

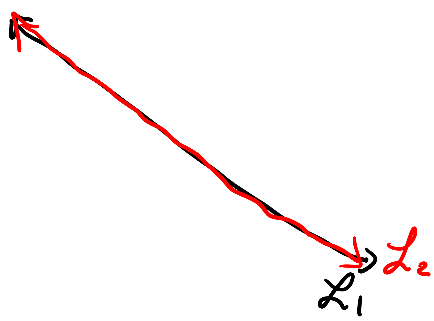
Find a row which can be removed to the augmented matrix to make a new system with two equations that has infinitely many solutions. Justify.

We notice that $\mathcal{L}_1$ and $\mathcal{L}_3$ are identical lines because they are multiples of one another. $\therefore$ If we remove the row associated to $\mathcal{L}_2$ then the system will have ∞-many solutions.

$$\therefore \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix}$$

Question 4. *(2 marks)* Illustrate and describe in terms of slope and intercept **all** relative positions of lines in a consistent linear system consisting of two lines.





unique solution when the two lines different slopes

∞-many solutions when both lines have the same slope and same intercept.

Question 5. *(2 marks)* Find the linear equation whose solution set it $(x, y, z) = (4, 0, 0) + s(2, 1, 0) + t(3, 0, 1)$ where $s, t \in \mathbb{R}$.

$x = 4 + 2s + 3t$ where $y = s$, $z = t$

$x = 4 + 2y + 3z$

$x - 2y - 3z = 4$