

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531***. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. (4 marks) Let A be an $n \times n$ matrix such that $A^2 + 2A - 4I_n = 0$. Show that $A - I_n$ is invertible, and find $(A - I_n)^{-1}$ in terms of A and I_n .

Question 2. (5 marks) Solve for the matrix A in the following equation:

$$\begin{bmatrix} -1 & 1 \\ -2 & 3 \end{bmatrix} \left(\begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} + 3(A^{-1})^T \right)^{-1} = A^T$$

Question 3. (3 marks) Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

If A and B are row equivalent matrices, then the linear systems $A\mathbf{x} = \mathbf{0}$ and $B\mathbf{x} = \mathbf{0}$ have the same solution set.

Question 4. (5 marks) If $B = \begin{bmatrix} 4 & -4 & 8 \\ 9 & 6 & -3 \\ 2 & 3 & -2 \end{bmatrix}$ is obtained from the 3×3 matrix A using the following elementary row operations:

- (a) Add $\frac{1}{4}$ of the first row to the 3rd row.
- (b) Multiply the 2nd row by $\frac{1}{3}$.
- (c) Interchange the first and 3rd row.

Find the matrix A and U where $A = UB$.