Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S5: Fall 2024: **Quiz 3** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(4 marks)* Let A be an $n \times n$ matrix such that $A^2 + 2A - 4I_n = 0$. Show that $A - I_n$ is invertible, and find $(A - I_n)^{-1}$ in terms of A and I_n.

$$A^2 + 2A - 4I = 0$$
$$A^2 + 2A - 3I = I$$
$$(A-I)(A+3I) = I \quad \text{also} \quad (A+3I)(A-I) = I$$

$\therefore$ $A - I$ is invertible and $(A-I)^{-1} = A + 3I$

Question 2. *(5 marks)* Solve for the matrix A in the following equation:

$$\underbrace{\begin{bmatrix} -1 & 1 \\ -2 & 3 \end{bmatrix}}_{B} \left(\underbrace{\begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}}_{C} + 3(A^{-1})^T \right)^{-1} = A^T$$

$$B(C + 3(A^{-1})^T)^{-1} = A^T$$
$$B^{-1}B(C + 3(A^{-1})^T)^{-1} = B^{-1}A^T$$
$$I\,(C + 3(A^{-1})^T)^{-1} = B^{-1}A^T$$
$$((C + 3(A^{-1})^T)^{-1})^{-1} = (B^{-1}A^T)^{-1}$$
$$C + 3(A^{-1})^T = (A^T)^{-1}(B^{-1})^{-1}$$
$$C + 3(A^{-1})^T = (A^{-1})^T B$$
$$C = (A^{-1})^T B - 3(A^{-1})^T$$
$$C = (A^{-1})^T (B - 3I)$$
$$C(B-3I)^{-1} = (A^{-1})^T (B-3I)(B-3I)^{-1}$$
$$C(B-3I)^{-1} = (A^{-1})^T I$$
$$C(B-3I)^{-1} = (A^T)^{-1}$$
$$(C(B-3I)^{-1})^{-1} = A^T$$
$$(B-3I)C^{-1} = A^T$$
$$((B-3I)C^{-1})^T = (A^T)^T$$
$$(C^{-1})^T(B-3I)^T = A$$
$$A = \left(\frac{1}{1} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \right)^T \begin{bmatrix} -4 & 1 \\ -2 & 0 \end{bmatrix}^T$$
$$= \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} -4 & -2 \\ 1 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} -4 & -2 \\ -7 & -4 \end{bmatrix}$$

Question 3. *(3 marks)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

If A and B are row equivalent matrices, then the linear systems $A\mathbf{x} = \mathbf{0}$ and $B\mathbf{x} = \mathbf{0}$ have the same solution set.

True, Since A and B are row equivalent : $A \sim k$ elem. row op. $\sim B$. If we perform Gauss Jordan on $[B|0] \sim l$ elem. row op. $\sim [R|0]$.

So $[A|0] \sim k$ elem. row op. $\sim [B|0] \sim l$ elem. row op $\sim [R|0]$

$\therefore$ A and B have the same solution set

Question 4. *(5 marks)* If $B = \begin{bmatrix} 4 & -4 & 8 \\ 9 & 6 & -3 \\ 2 & 3 & -2 \end{bmatrix}$ is obtained from the 3×3 matrix A using the following elementary row operations:

(a) Add $\frac{1}{4}$ of the first row to the 3rd row.

(b) Multiply the 2nd row by $\frac{1}{3}$.

(c) Interchange the first and 3rd row.

Find the matrix A and U where $A = UB$.

$I_3 \sim \frac{1}{4}R_1 + R_3 \to R_3$ E_1

$I_3 \sim \frac{1}{3}R_2 \to R_2$ E_2

$I_3 \sim R_1 \leftrightarrow R_3$ E_3

And we have $E_3E_2E_1A = B$

$(E_3E_2E_1)^{-1}E_3E_2E_1A = (E_3E_2E_1)^{-1}B$

$A = E_1^{-1}E_2^{-1}E_3^{-1}B$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1/4 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 4 & -4 & 8 \\ 9 & 6 & -3 \\ 2 & 3 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1/4 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 4 & -4 & 8 \\ 9 & 6 & -3 \\ 2 & 3 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & -\frac{1}{4} \end{bmatrix}\begin{bmatrix} 4 & -4 & 8 \\ 9 & 6 & -3 \\ 2 & 3 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 & -2 \\ 27 & 18 & -9 \\ 7/2 & -\frac{15}{4} & 17/2 \end{bmatrix}$$

and $U = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & -1/4 \end{bmatrix}$