Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S5: Fall 2024: **Quiz 4** name: ____________________

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1.

a. *(3 marks)* Determine all s and t such that the given matrix is symmetric

$$\begin{bmatrix} s & 2s & st \\ t & -1 & s \\ t & s^2 & s \end{bmatrix}$$

b. *(2 marks)* If A and B are invertible symmetric matrices such that A and B commute show that $A^{-1}B^{-1}$ is also invertible and symmetric.

Question 2. *(5 marks)* Solve for x where

$$\begin{vmatrix} \sin x & \cos x \\ -\cos x & \sin x \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ e^x & 1 & e^x \\ 1 & e^x & 0 \end{vmatrix}$$

Question 3. *(3 marks)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

Let $A\mathbf{x} = \mathbf{b}$ be any consistent system of linear equations, and let $\mathbf{x}_1$ be a fixed solution. Then every solution to the system can be written in the form $\mathbf{x} = \mathbf{x}_1 + \mathbf{x}_0$, where $\mathbf{x}_0$ is a solution to $A\mathbf{x} = \mathbf{0}$.

Question 4. *(5 marks)* Let A be a 3×3 matrix such that $\det(A) = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 2$. Find the following: $\begin{vmatrix} a+b & 3d+3e & g+h \\ 2a-3b & 6d-9e & 2g-3h \\ 2c & 6f & 2i \end{vmatrix}$.

Bonus. *(3 marks)* Let A and B be $m \times n$ and $n \times m$ matrices, respectively. If $m > n$, show that AB is not invertible.