Question 1. *(5 marks)* Let A and B be two 3×3 matrices such that $\det(A) = 2$ and $\det(B) = -4$. Find the following $\det\left(5B^{-1}A + \mathrm{adj}(A^{-1}B)\right)$.

Question 2. *(5 marks)* Given the vertices $A(2, -2, 4)$, $B(4, -1, 1)$, and $C(3, -1, 2)$ of a triangle. Only using vectors find the components of the vector **u** with initial point being the midpoint of the side AB and terminal point being the midpoint of the side AC.

Question 3. *(5 marks)* Let $\mathbf{u}$ and $\mathbf{v}$ be vectors in $\mathbb{R}^3$ such that $||\mathbf{u}|| = 4$, $||\mathbf{v}|| = \sqrt{3}$ and $\mathbf{u} \cdot \mathbf{v} = -6$. For which values of t, if any, is the angle between $\mathbf{u} + \mathbf{v}$ and $\mathbf{u} + t\mathbf{v}$ acute. (An angle θ is said to be *acute* if $0 < \theta < \frac{\pi}{2}$).

Question 4. *(3 marks)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

If A is an $n \times n$ skew-symmetric matrix, such that n is odd, then $A\mathbf{x} = \mathbf{0}$ has nontrivial solutions. (A matrix is skew-symmetric if $A^T = -A$.)