Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S5: Fall 2024: **Quiz 5** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(5 marks)* Let A and B be two 3×3 matrices such that $\det(A) = 2$ and $\det(B) = -4$. Find the following $\det\left(5B^{-1}A + \operatorname{adj}(A^{-1}B)\right)$.

$$(A^{-1}B)^{-1} = \frac{1}{\det(A^{-1}B)}\operatorname{adj}(A^{-1}B)$$

$$B^{-1}(A^{-1})^{-1} = \frac{1}{\det(A^{-1})\det B}\operatorname{adj}(A^{-1}B)$$

$$\det(A^{-1})\det B\ B^{-1}A = \operatorname{adj}(A^{-1}B)$$

$$\frac{\det B}{\det A}B^{-1}A = \operatorname{adj}(A^{-1}B)$$

$$\frac{-4}{2}B^{-1}A = \operatorname{adj}(A^{-1}B)$$

$$-2B^{-1}A = \operatorname{adj}(A^{-1}B)$$

$$= \det(5B^{-1}A - 2B^{-1}A)$$

$$= \det(3B^{-1}A)$$

$$= 3^3\det(B^{-1}A)$$

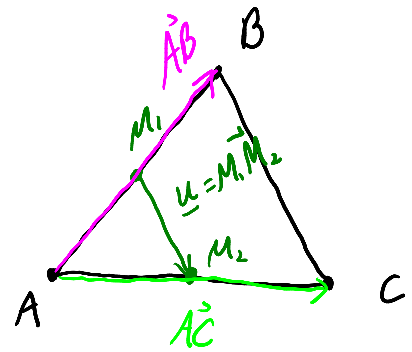
$$= 3^3\det(B^{-1})\det A$$

$$= 3^3\frac{1}{\det B}\det A$$

$$= 3^3\frac{1}{-4}\cdot 2$$

$$= -\frac{3^3}{2}$$

Question 2. *(5 marks)* Given the vertices $A(2,-2,4)$, $B(4,-1,1)$, and $C(3,-1,2)$ of a triangle. Only using vectors find the components of the vector $\mathbf{u}$ with initial point being the midpoint of the side AB and terminal point being the midpoint of the side AC.



$$\vec{AM_1} + \vec{M_1M_2} + \vec{M_2A} = \underline{0}$$

$$\vec{M_1M_2} = -\vec{AM_1} - \vec{M_2A}$$

$$\underline{u} = -\frac{1}{2}\vec{AB} - \left(-\frac{1}{2}\vec{AC}\right)$$

$$\underline{u} = \frac{1}{2}\vec{BA} + \frac{1}{2}\vec{AC}$$

$$= \frac{1}{2}(-2,-1,3) + \frac{1}{2}(1,1,-2)$$

$$= \left(-\frac{1}{2}, 0, \frac{1}{2}\right)$$

$$\vec{BA} = \vec{OA} - \vec{OB} = (2,-2,4) - (4,-1,1)$$
$$= (-2,-1,3)$$

$$\vec{AC} = \vec{OC} - \vec{OA} = (3,-1,2) - (2,-2,4)$$
$$= (1,1,-2)$$

Question 3. *(5 marks)* Let **u** and **v** be vectors in $\mathbb{R}^3$ such that $||\mathbf{u}|| = 4$, $||\mathbf{v}|| = \sqrt{3}$ and $\mathbf{u} \cdot \mathbf{v} = -6$. For which values of t, if any, is the angle between $\mathbf{u}+\mathbf{v}$ and $\mathbf{u}+t\mathbf{v}$ acute. (An angle θ is said to be *acute* if $0 < \theta < \frac{\pi}{2}$).

$$(\underline{u}+\underline{v}) \cdot (\underline{u}+t\underline{v}) = \underline{u}\cdot\underline{u} + \underline{u}\cdot(t\underline{v}) + \underline{v}\cdot\underline{u} + \underline{v}\cdot(t\underline{v})$$
$$= \|\underline{u}\|^2 + t\,\underline{u}\cdot\underline{v} + \underline{u}\cdot\underline{v} + t\,\underline{v}\cdot\underline{v}$$
$$= 4^2 + (t+1)\,\underline{u}\cdot\underline{v} + t\|\underline{v}\|^2$$
$$= 16 + (t+1)(-6) + t(\sqrt{3})^2$$
$$= 16 + (t+1)(-6) + 3t$$
$$= 16 - 6t - 6 + 3t$$
$$= 10 - 3t > 0$$ to be acute
$$10 > 3t$$
$$\frac{10}{3} > t$$

$\therefore$ if $t > \frac{10}{3}$ the angle is acute.

Question 4. *(3 marks)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

If A is an $n \times n$ skew-symmetric matrix, such that n is odd, then $A\mathbf{x} = \mathbf{0}$ has nontrivial solutions. (A matrix is skew-symmetric if $A^T = -A$.)

True,

Since A is skew symmetric $A^T = -A$
$$\det(A^T) = \det(-A)$$
$$\det(A) = (-1)^n \det(A)$$
$$\det(A) = -\det(A)$$ since n is odd
$$2\det(A) = 0$$
$$\det(A) = 0$$

$\therefore$ by equivalence theorem $A\underline{x} = \underline{0}$ has non-trivial solutions.