Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S5: Fall 2024: **Quiz 6** name: ______________________

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. Consider the lines $(L): \begin{cases} x = 1+2t \\ y = 3-t \\ z = 5+t \end{cases}$, $(L'): \begin{cases} x = 1-4t \\ y = 1+2t \\ z = 3-2t \end{cases}$, $t \in \mathbb{R}$, the plane $(P): x-2y+z+5=0$, and the point $A(4,-2,5)$

a. *(5 marks)* Using projections find the distance between the lines (L) and (L').

b. *(1 mark)* Find the parametric equation of the line orthogonal to (P) and that passes through A.

c. *(4 marks)* Find the point on the plane (P) closest to the point A.

d. *(1 mark)* Find the parametric equation of the plane that contains (L) and (L').

Question 2. Consider the planes $\mathscr{P}_1 : x+y+z=b_1$ and $\mathscr{P}_2 : x+2y+cz=b_2$ where b_1, b_2, c are fixed values, $P(1,1,1)$ is a point of the intersection between $\mathscr{P}_1$ and $\mathscr{P}_2$ and the intersection is parallel to $\mathbf{u}=(2,-1,-1)$.

a. *(2 marks)* What kind of geometrical object is the intersection of $\mathscr{P}_1$ and $\mathscr{P}_2$, justify.

b. *(3 marks)* Solve for c, as part of your justification use a sketch and state any appropriate theorems.

Question 3. *(3 marks)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

If $\mathbf{a}$ and $\mathbf{b}$ are nonzero orthogonal vectors, then for every nonzero vector $\mathbf{u}$, we have $\text{proj}_{\mathbf{a}}(\text{proj}_{\mathbf{b}}(\mathbf{u})) = \mathbf{0}$.

Bonus Question. *(2 marks)* What kind of geometrical object is $r(t)=(t^4,t^4,t^4)$ where $t \in \mathbb{R}$.