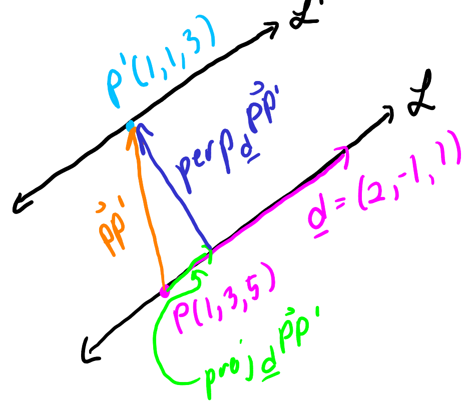
Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S5: Fall 2024: **Quiz 6** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. Consider the lines $(L):\begin{cases} x = 1+2t \\ y = 3-t \\ z = 5+t \end{cases}$, $(L'):\begin{cases} x = 1-4t \\ y = 1+2t \\ z = 3-2t \end{cases}$, $t \in \mathbb{R}$, the plane $(P): x-2y+z+5=0$, and the point $A(4,-2,5)$

a. *(5 marks)* Using projections find the distance between the lines (L) and (L').



Note that $\mathcal{L}$ and $\mathcal{L}'$ are $\parallel$ since $\underline{d} \parallel \underline{d}'$ because $\underline{d}' = -2\underline{d}$

$$\text{distance} = \|\text{perp}_{\underline{d}}\vec{PP'}\| = \|(0,2,2)\| = \sqrt{0^2+2^2+2^2} = \sqrt{8}$$

$$\vec{PP'} = \vec{OP'} - \vec{OP} = (1,3,5)-(1,1,3) = (0,2,2)$$

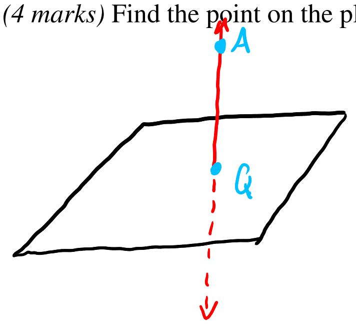
$$\text{proj}_{\underline{d}}\vec{PP'} = \frac{\vec{PP'}\cdot\underline{d}}{\underline{d}\cdot\underline{d}}\underline{d} = \frac{(0,2,2)\cdot(2,-1,1)}{(0,2,2)\cdot(0,2,2)}(0,2,2) = \underline{0}$$

$\therefore$ $\text{perp}_{\underline{d}}\vec{PP'} = (0,2,2)$

b. *(1 mark)* Find the parametric equation of the line orthogonal to (P) and that passes through A.

$$\underline{x} = \underline{A} + t\underline{n} = (4,-2,5) + t(1,-2,1) \quad \text{where } t \in \mathbb{R}$$

c. *(4 marks)* Find the point on the plane (P) closest to the point A.



Closest point will be the intersection of the line found in part b and the plane

$$(4+t) - 2(-2-2t) + (5+t) + 5 = 0$$
$$4+t + 4 + 4t + 5 + t + 5 = 0$$
$$6t = -18$$
$$t = -3$$

The intersection happens when $t = -3$

$$\underline{x} = (4,-2,5) - 3(1,-2,1) = (1,4,2)$$

$\therefore$ the closest point is $Q(1,4,2)$.

d. *(1 mark)* Find the parametric equation of the plane that contains (L) and (L').

$$\underline{x} = \underline{P_0} + s\underline{d_1} + t\underline{d_2} = \underline{P} + s\underline{d} + t\vec{PP'} \quad s,t \in \mathbb{R}$$
$$= (1,3,5) + s(2,-1,1) + t(0,2,2)$$

Question 2. Consider the planes $\mathscr{P}_1 : x+y+z=b_1$ and $\mathscr{P}_2 : x+2y+cz=b_2$ where b_1, b_2, c are fixed values, $P(1,1,1)$ is a point of the intersection between $\mathscr{P}_1$ and $\mathscr{P}_2$ and the intersection is parallel to $\mathbf{u}=(2,-1,-1)$.

a. *(2 marks)* What kind of geometrical object is the intersection of $\mathscr{P}_1$ and $\mathscr{P}_2$, justify.

$P_1 \nparallel P_2$ since $\underline{n}_1 \nparallel \underline{n}_2$ because $\nexists\, k$ s.t. $\underline{n}_1 = k\underline{n}_2$.

$\therefore$ the intersection is a line.

b. *(3 marks)* Solve for c, as part of your justification use a sketch and state any appropriate theorems.

By a theorem seen in class we know that the solution set of the homogeneous system is orthogonal to the rows of the coefficient matrix.

$\therefore \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & c \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ the solution set is orthogonal to the normals $\underline{n}_1=(1,1,1)$ and $\underline{n}_2=(1,2,c)$

Also the solution set of the homogeneous system is a line passing through the origin and parallel to $\underline{u}$ $\therefore \underline{x} = t(2,-1,-1)$ $t \in \mathbb{R}$

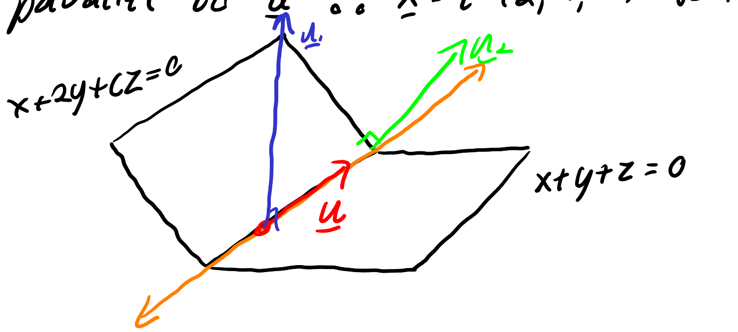
$\therefore \underline{n}_2 \cdot \underline{x} = 0$

$(1,2,c)\cdot(2t,-t,-t)=0$

$2t-2t-ct=0$

$ct=0 \quad \forall t \in \mathbb{R}$

$\therefore c=0$



Question 3. *(3 marks)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

If $\mathbf{a}$ and $\mathbf{b}$ are nonzero orthogonal vectors, then for every nonzero vector $\mathbf{u}$, we have $\text{proj}_{\mathbf{a}}(\text{proj}_{\mathbf{b}}(\mathbf{u}))=\mathbf{0}$.

True,

$$\text{proj}_{\underline{a}}\left(\frac{\underline{u}\cdot\underline{b}}{\underline{b}\cdot\underline{b}}\underline{b}\right) = \frac{\left(\frac{\underline{u}\cdot\underline{b}}{\underline{b}\cdot\underline{b}}\underline{b}\right)\cdot\underline{a}}{\underline{a}\cdot\underline{a}}\underline{a}$$

$$= \frac{\left(\frac{\underline{u}\cdot\underline{b}}{\underline{b}\cdot\underline{b}}\right)(\underline{b}\cdot\underline{a})}{\underline{a}\cdot\underline{a}}\underline{a}$$

$$= \frac{\left(\frac{\underline{u}\cdot\underline{b}}{\underline{b}\cdot\underline{b}}\right)0}{\underline{a}\cdot\underline{a}}\underline{a}$$

$$= \underline{0}$$

Bonus Question. *(2 marks)* What kind of geometrical object is $r(t)=(t^4,t^4,t^4)$ where $t\in\mathbb{R}$.