

Question 1. Consider the points $A(2, -2, 4)$, $B(4, -1, 1)$, and $C(3, -1, 2)$.

a. (3 marks) Find the area of the triangle ABC .

b. (3 marks) Find the exact value of the tangent of the angle at the vertex A of the triangle ABC .

Question 2. (5 marks) Given that the following lines $(L_1): \begin{cases} x = 3 + t \\ y = 5 + t \\ z = 2 + t \end{cases}$, and $(L_2): \begin{cases} x = -1 + 2s \\ y = 1 + s \\ z = s \end{cases}$, $t, s \in \mathbb{R}$ are skew lines. Find the equation of the line that intersects (L_1) and (L_2) at right angles.

Question 3. (3 marks) Show that the additive inverse of any vector in a vector space is unique. *Show every step, justify every step, and cite the axiom(s) used!!!*

Question 4. (4 marks) Let $W = \left\{ A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_{22} \mid AB = BA \right\}$ where $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. Determine whether W is a subspace of M_{22} .

Question 5. (2 marks) Determine whether $U = \{(r, 0, s) \mid rs = 0 \text{ and } r, s \in \mathbb{R}\}$ is a vector space with the usual operations in \mathbb{R}^3 .