Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S6: Fall 2024: **Quiz 1** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(3 marks each)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

a. Consider a system of linear equations with augmented matrix A. If A has a row of zeros, there is more than one solution.

False, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ is the augmented matrix for $\begin{cases} x = 0 \\ 0 = 1 \end{cases}$

$\therefore$ the system is inconsistent

b. If each equation in a consistent linear system is multiplied through by a constant c, then all solutions to the new system can be obtained by multiplying solutions from the original system by c.

False, $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ is the augmented matrix for $\begin{cases} x = 1 \\ y = 1 \end{cases}$

and if $c = 2$ $\begin{bmatrix} 2 & 0 & 2 \\ 0 & 2 & 2 \end{bmatrix}$ is the augmented matrix for $\begin{cases} 2x = 2 \\ 2y = 2 \end{cases} \Rightarrow \begin{cases} x = 1 \\ y = 1 \end{cases}$

the solution set is preserved when elem. row op. are applied

Question 2. *(3 marks)* Find (if possible) conditions on a and b such that the system has no solution, one solution, and infinitely many solutions. Justify.

$$\begin{cases} x & +ay = 1 \\ 2x & +by = 3 \end{cases} \Rightarrow \begin{cases} x = -ay + 1 \\ x = -\frac{b}{2}y + \frac{3}{2} \end{cases}$$

note that both lines have different intercept.
$\therefore$ impossible to have identical lines
$\therefore \nexists\, a, b$ s.t. there are ∞ many solutions

If $-a \neq -\frac{b}{2} \Rightarrow 2a \neq b$ then the slopes are different and both lines intersect at a point. $\therefore$ unique solution.

If $2a = b$ then the slopes are the same with different intercept and both lines do not intersect.
$\therefore$ no solutions.

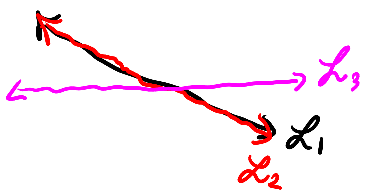
Question 3. *(2 marks)* Consider the following augmented matrix of a consistent linear system.

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix} \Rightarrow \begin{array}{l} \mathcal{L}_1: x+2y=3 \\ \mathcal{L}_2: 2x+4y=6 \end{array}$$

Two identical lines, i.e. ∞ many solutions

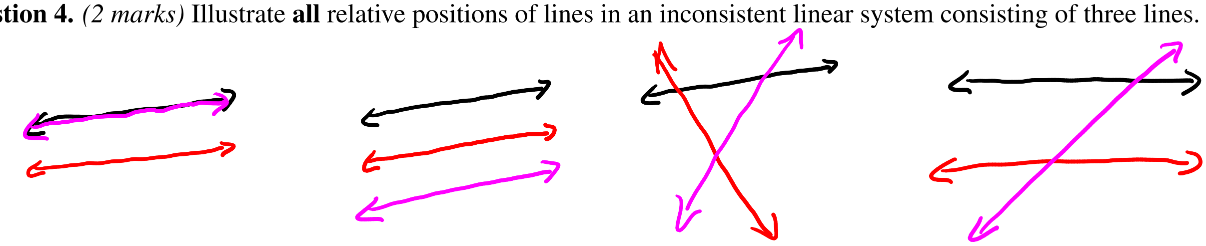
Find a row which can be added to the augmented matrix to make a new system with three equations that has a unique solution. Justify.

If we add a line $\mathcal{L}_3$ which is not identical to $\mathcal{L}_1$ and $\mathcal{L}_2$. Then we obtain a single intersection between $\mathcal{L}_1$ and $\mathcal{L}_2$ i.e. a unique solution. Lets add $\mathcal{L}_3: y=1$



$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 0 & 1 & 1 \end{bmatrix}$$ has a unique solution.

Question 4. *(2 marks)* Illustrate **all** relative positions of lines in an inconsistent linear system consisting of three lines.



Question 5. *(3 marks)* Show that a system consisting of exactly one linear equation can have no solution, one solution, or infinitely many solutions. Give examples.

No solution: $0x+0y=1$ No values of x and y can satisfy the equation.

One solution: $x=1$ Unique solution $x=1$

∞-many solutions: $x+y=0$ All points that are on the graph of the line are solutions.