Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S6: Fall 2024: **Quiz 2** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(3 marks)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

If $AB+BA$ is defined, then A and B are square matrices of the same size.

True, since $AB+BA$ is defined AB and BA have the same dimension, say $m\times n$.
since AB is defined and has dimension $m\times n$ it implies that A has m rows and B has n col.

since BA is defined and has dimension $m\times n$ it implies that A has n col and B has m rows

In addition #col of B = # rows of A $\therefore m=n$

$\therefore$ A and B are $n\times n$ matrices.

Question 2. *(6 marks)* Consider the system

$$\begin{array}{rcrcrcl} 2kx & + & (k+1)y & & & = & 2 \\ x & + & y & + & z & = & 0 \\ kx & + & (2k-1)y & & & = & 1 \end{array}$$

Find the value(s) of k, if any, such that the system has:

a. no solutions, justify.

b. a unique solution, justify.

c. infiniely many solutions, justify.

$$\begin{bmatrix} 2k & k+1 & 0 & 2 \\ 1 & 1 & 1 & 0 \\ k & 2k-1 & 0 & 1 \end{bmatrix} \sim_{R_1\leftrightarrow R_2} \begin{bmatrix} 1 & 1 & 1 & 0 \\ 2k & k+1 & 0 & 2 \\ k & 2k-1 & 0 & 1 \end{bmatrix}$$

$$\sim \begin{array}{l} -2kR_1+R_2\to R_2 \\ -kR_1+R_3\to R_3 \end{array} \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1-k & -2k & 2 \\ 0 & k-1 & -k & 1 \end{bmatrix}$$

$$\sim_{R_2+R_3\to R_3} \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1-k & -2k & 2 \\ 0 & 0 & -3k & 3 \end{bmatrix} \sim_{-\frac{1}{3}R_3\to R_3} \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1-k & -2k & 2 \\ 0 & 0 & k & -1 \end{bmatrix}$$

If $k=0$ then $\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & -1 \end{bmatrix}$ leading entry in constant column, so no solution.

If $k=1$ then $\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & -2 & 2 \\ 0 & 0 & 1 & -1 \end{bmatrix} \sim_{\frac{1}{2}R_2+R_3\to R_3} \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & -2 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ #leading entries in var. < # var, so ∞-many solutions.

If $k\neq 0,1$ then #leading entries in var col = # var, so unique solution.

Question 3. *(3 marks)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

If A and B are square matrices of the same order, then $(AB)^T = A^T B^T$.

False, $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 3 \\ 1 & 4 \end{bmatrix}$ $\quad (AB)^T = \left(\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} -2 & 3 \\ 1 & 4 \end{bmatrix} \right)^T = \left(\begin{bmatrix} 0 & 11 \\ -5 & 24 \end{bmatrix} \right)^T = \begin{bmatrix} 0 & -5 \\ 11 & 24 \end{bmatrix}$

$A^T B^T = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ 3 & 4 \end{bmatrix} = [10$

So $(AB)^T \neq B^T A^T$

Question 3. *(6 marks)* Find **all** matrices A such that $A \begin{bmatrix} 2 & 3 \\ 5 & 7 \\ -2 & -3 \end{bmatrix} - \begin{bmatrix} 1 \\ 2 \end{bmatrix}^T = \text{trace}\left(\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \right) \begin{bmatrix} 1 \\ 2 \end{bmatrix}^T$.

$A \begin{bmatrix} 2 & 3 \\ 5 & 7 \\ -2 & -3 \end{bmatrix} - \begin{bmatrix} 1 \\ 2 \end{bmatrix}^T = 0 \begin{bmatrix} 1 \\ 2 \end{bmatrix}^T \Rightarrow A \begin{bmatrix} 2 & 3 \\ 5 & 7 \\ -2 & -3 \end{bmatrix} - \begin{bmatrix} 1 \\ 2 \end{bmatrix}^T = \begin{bmatrix} 0 & 0 \end{bmatrix} \quad A \begin{bmatrix} 2 & 3 \\ 5 & 7 \\ -2 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \end{bmatrix}$

Let $A = \begin{bmatrix} x & y & z \end{bmatrix}$ $\quad \begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 5 & 7 \\ -2 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \end{bmatrix}$

$\begin{bmatrix} 2x + 5y - 2z & 3x + 7y - 3z \end{bmatrix} = \begin{bmatrix} 1 & 2 \end{bmatrix}$

So $2x + 5y - 2z = 1$
$3x + 7y - 3z = 2$

$\begin{bmatrix} 2 & 5 & -2 & 1 \\ 3 & 7 & -3 & 2 \end{bmatrix} \sim_{2R_2 \to R_2} \begin{bmatrix} 2 & 5 & -2 & 1 \\ 6 & 14 & -6 & 4 \end{bmatrix} \sim_{-3R_1 + R_2 \to R_2} \begin{bmatrix} 2 & 5 & -2 & 1 \\ 0 & -1 & 0 & 1 \end{bmatrix}$

$\sim \begin{matrix} 5R_2 + R_1 \to R_1 \\ -R_2 \to R_2 \end{matrix} \begin{bmatrix} 2 & 0 & -2 & 6 \\ 0 & 1 & 0 & -1 \end{bmatrix}$

$\sim_{\frac{1}{2}R_1 \to R_1} \begin{bmatrix} 1 & 0 & -1 & 3 \\ 0 & 1 & 0 & -1 \end{bmatrix}$

Let $z = t$, $t \in \mathbb{R}$
$\Rightarrow x = 3 + t$
$y = -1$

$\therefore A = \begin{bmatrix} 3+t & -1 & t \end{bmatrix}$, $t \in \mathbb{R}$