Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S6: Fall 2024: **Quiz 3** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(4 marks)* Prove: If the reduced row echelon form of A is I_n, and E is an $n \times n$ elementary matrix, then the system $EA^2A^T\mathbf{x} = \mathbf{0}$ has only the trivial solution.

A is invertible by the equivalence thm since its RREF is I.

E an elementary matrix is invertible

A^T is invertible since A is invertible.

$EA^2A^T = EAAE^T$ is invertible since a product of invertible matrices

$\therefore$ by the equivalence thm $EA^2A^T\underline{x} = \underline{0}$ has only the trivial solution.

Question 2. *(5 marks)* Solve for the matrix A in the following equation:

$$\underbrace{\begin{bmatrix} -1 & 1 \\ -2 & 3 \end{bmatrix}}_{B} \left(\underbrace{\begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}}_{C} + 3(A^{-1})^T \right)^{-1} = A^T$$

$$B(C + 3(A^{-1})^T)^{-1} = A^T$$

$$B^{-1}B(C + 3(A^{-1})^T)^{-1} = B^{-1}A^T$$

$$I\ (C + 3(A^{-1})^T)^{-1} = B^{-1}A^T$$

$$((C + 3(A^{-1})^T)^{-1})^{-1} = (B^{-1}A^T)^{-1}$$

$$C + 3(A^{-1})^T = (A^T)^{-1}(B^{-1})^{-1}$$

$$C + 3(A^{-1})^T = (A^{-1})^T B$$

$$C = (A^{-1})^T B - 3(A^{-1})^T$$

$$C = (A^{-1})^T (B - 3I)$$

$$C(B-3I)^{-1} = (A^{-1})^T (B-3I)(B-3I)^{-1}$$

$$C(B-3I)^{-1} = (A^{-1})^T I$$

$$C(B-3I)^{-1} = (A^T)^{-1}$$

$$(C(B-3I)^{-1})^{-1} = A^T$$

$$(B-3I)C^{-1} = A^T$$

$$((B-3I)C^{-1})^T = (A^T)^T$$

$$(C^{-1})^T(B-3I)^T = A$$

$$A = \left(\frac{1}{1}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\right)^T \begin{bmatrix} -4 & 1 \\ -2 & 0 \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} -4 & -2 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & -2 \\ -7 & -4 \end{bmatrix}$$

Question 3. *(3 marks)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

If A is an invertible matrix and B is row equivalent to A, then B is also invertible.

True,
since B is row equivalent to A: $B \sim$ k elem. row op $\sim A$.
And if we perform Gauss-Jordan on $A \sim$ l elem. row op. $\sim I$ since A is invertible
$\therefore$ $B \sim$ k elem. row op. $\sim A \sim$ l elem row op $\sim I$
$\therefore$ B is invertible by the equivalence thm since its RREF is I.

Question 4. *(5 marks)* Given $A = \begin{bmatrix} 0 & 2 & 0 \\ 1 & 0 & 0 \\ 0 & 3 & 0 \end{bmatrix}$. Find an invertible matrix U such that $A = UR$ where R is the reduced row echelon form of A.

$$A \sim_{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 3 & 0 \end{bmatrix}$$

$$\sim_{\frac{1}{2}R_2 \to R_2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 0 \end{bmatrix}$$

$$\sim_{-3R_2 + R_3 \to R_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = R$$

$I_3 \sim R_1 \leftrightarrow R_2$ E_1

$I_3 \sim \frac{1}{2}R_2 \to R_2$ E_2

$I_3 \sim -3R_2 + R_3 \to R_3$ E_3

So $E_3E_2E_1A = R$

$$(E_3E_2E_1)^{-1}E_3E_2E_1A = (E_3E_2E_1)^{-1}R$$
$$IA = E_1^{-1}E_2^{-1}E_3^{-1}R$$
$$A = E_1^{-1}E_2^{-1}E_3^{-1}R$$

$$A = E_1^{-1}E_2^{-1}E_3^{-1}R$$
$$= \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}R$$
$$= \begin{bmatrix} 0 & 2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 3 & 1 \end{bmatrix}R$$
$$= \begin{bmatrix} 0 & 2 & 0 \\ 1 & 0 & 0 \\ 0 & 3 & 1 \end{bmatrix}R$$