Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S6: Fall 2024: **Quiz 4** name: ____________

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1.

a. *(3 marks)* Find all symmetric 2×2 matrices A such that $A^2 = 0$.

b. *(2 marks)* If A is invertible and skew-symmetric ($A^T = -A$), the same is true of A^{-1}

Question 2. *(5 marks)* Given the following matrices:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 5 & 0 & 1 \end{bmatrix}, \; B = \begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 1 & -1 \end{bmatrix} \text{ and } C = \begin{bmatrix} 2 & -2 & 6 \\ 0 & 1 & -3 \end{bmatrix}$$

Find $\det(X)$ given that X satisfies the equation

$$(X + BC)^{-1} = A.$$

Question 3. *(3 marks)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

Let A be an $n \times n$ matrix and S is an $n \times n$ invertible matrix. If $\mathbf{x}$ is a solution to the system $(S^{-1}AS)\mathbf{x} = \mathbf{b}$, then $S\mathbf{x}$ is a solution to the system $A\mathbf{y} = S\mathbf{b}$.

Question 4. *(5 marks)* Find the real numbers x and such that

$$0 = \begin{vmatrix} 1 & x & x^2 & x^3 \\ x & x^2 & x^3 & 1 \\ x^2 & x^3 & 1 & x \\ x^3 & 1 & x & x^2 \end{vmatrix}$$

Bonus. *(3 marks)* Let A and B be $m \times n$ and $n \times m$ matrices, respectively. If $m > n$, show that AB is not invertible.