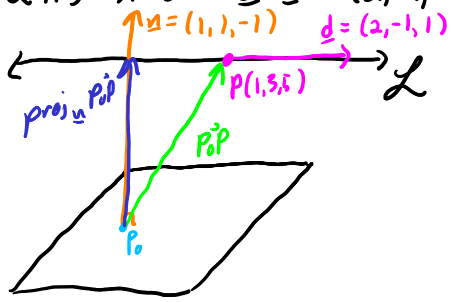
Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S6: Fall 2024: **Quiz 6** name: ____________

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. Consider the line $(L): \begin{cases} x = 1+2t \\ y = 3-t \\ z = 5+t \end{cases}$ where $t \in \mathbb{R}$, the plane $(P): x+y-z+5=0$, and the point $A(4,-2,5)$

a. *(5 marks)* Using projections find the distance between the line (L) and the plane (P).

$\mathcal{L} \parallel P$ since $\underline{d} \cdot \underline{n} = (2,-1,1) \cdot (1,1,-1) = 2-1-1 = 0.$



Let $x = y = 0$ $\quad 0+0-z+5=0$

$-z=-5$

$z=5$

$\therefore P_0(0,0,5)$ lies on the plane

$\vec{P_0P} = \vec{OP} - \vec{OP_0} = (1,3,5)-(0,0,5) = (1,3,0)$

$\text{proj}_{\underline{n}} \vec{P_0P} = \dfrac{\vec{P_0P} \cdot \underline{n}}{\underline{n} \cdot \underline{n}} \underline{n}$

$= \dfrac{(1,3,0)\cdot(1,1,-1)}{(1,1,-1)\cdot(1,1,-1)}(1,1,-1)$

$= \frac{4}{3}(1,1,-1)$

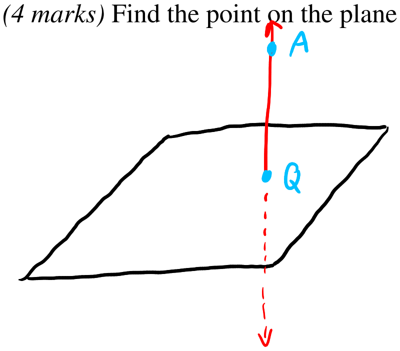
distance $= \|\text{proj}_{\underline{n}} \vec{P_0P}\| = \|\frac{4}{3}(1,1,-1)\|$

$= \frac{4}{3}\sqrt{1^2+1^2+(-1)^2} = \frac{4}{3}\sqrt{3}$

b. *(1 mark)* Find the parametric equation of a line orthogonal to (P) and that passes through A.

$\underline{x} = \underline{A} + t\underline{n} = (4,-2,5) + t(1,1,-1) \quad t \in \mathbb{R}$

c. *(4 marks)* Find the point on the plane (P) closest to the point A.



Closest point will be the intersection of the line found in part b and the plane

$(4+t)+(-2+t)-(5-t)+5=0$

$3t+2=0$

$t = -\frac{2}{3}$

$\therefore$ intersection happens when $t = -\frac{2}{3}$

$\underline{x} = (4,-2,5) - \frac{2}{3}(1,1,-1) = \left(\frac{10}{3}, \frac{-8}{3}, \frac{17}{3}\right)$

$\therefore$ the closest point is $Q\left(\frac{10}{3}, \frac{-8}{3}, \frac{17}{3}\right)$

d. *(1 mark)* Find the parametric equation of the plane that contains (L) and is orthogonal to (P).

$\underline{x} = \underline{P_0} + s\underline{d_1} + t\underline{d_2}$

$= \underline{P} + s\underline{d} + t\underline{n} \quad s,t \in \mathbb{R}$

$= (1,3,5) + s(2,-1,1) + t(1,1,-1)$

Question 2. *(5 marks)* Consider the planes $\mathscr{P}_1 : x+y+z = b_1$ and $\mathscr{P}_2 : x+2y+cz = b_2$ where b_1, b_2, c are fixed values, $P(1,1,1)$ is a point of the intersection between $\mathscr{P}_1$ and $\mathscr{P}_2$ and the intersection is parallel to $\mathbf{u} = (2,-1,-1)$.

a. *(2 marks)* What kind of geometrical object is the intersection of $\mathscr{P}_1$ and $\mathscr{P}_2$, justify.

$P_1 \nparallel P_2$ since $\underline{n_1} \nparallel \underline{n_2}$ because $\nexists\, k$ s.t. $\underline{n_1} = k\underline{n_2}$.

$\therefore$ the intersection is a line.

b. *(3 marks)* Find the equation of the intersection, as part of your justification use a sketch and state any appropriate theorems.

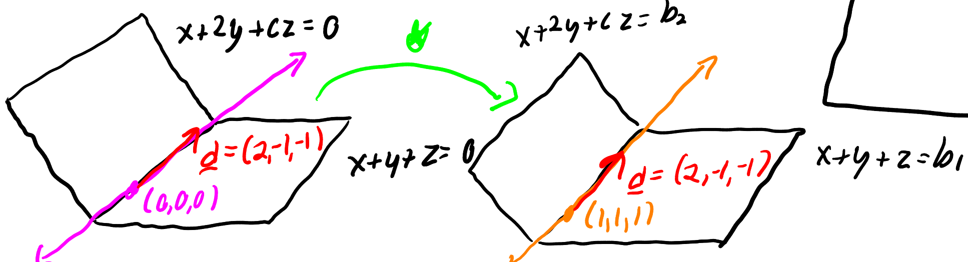
Since the intersection is a line, the solution set of the homogeneous system $\begin{cases} x+y+z=0 \\ x+2y+cz=0 \end{cases}$ is $\underline{x} = t\underline{u} = t(2,-1,-1) \quad t \in \mathbb{R}$

By a theorem seen in class we have

$$\begin{bmatrix} \text{sol. set of} \\ A\underline{x} = \underline{b} \end{bmatrix} = \begin{bmatrix} \text{particular sol.} \\ \text{of } A\underline{x} = \underline{b} \end{bmatrix} + \begin{bmatrix} \text{general sol.} \\ \text{of } A\underline{x} = \underline{0} \end{bmatrix}$$

$$= (1,1,1) + t(2,-1,-1) \quad t \in \mathbb{R}$$

Translations of the planes, so translation of the solution set.



Question 3. *(3 marks)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

If $\mathbf{a}$ and $\mathbf{u}$ are nonzero vectors, then $\text{proj}_{\mathbf{a}}(\text{proj}_{\mathbf{a}}(\mathbf{u})) = \text{proj}_{\mathbf{a}}(\mathbf{u})$.

true,

$$\text{proj}_{\underline{a}}(\text{proj}_{\underline{a}}\underline{u}) = \text{proj}_{\underline{a}}\left(\frac{\underline{a}\cdot\underline{u}}{\underline{a}\cdot\underline{a}}\underline{a}\right)$$

$$= \frac{\left(\frac{\underline{a}\cdot\underline{u}}{\underline{a}\cdot\underline{a}}\underline{a}\right)\cdot\underline{a}}{\underline{a}\cdot\underline{a}}\underline{a}$$

$$= \frac{\left(\frac{\underline{a}\cdot\underline{u}}{\underline{a}\cdot\underline{a}}\right)\cancel{\underline{a}\cdot\underline{a}}}{\cancel{\underline{a}\cdot\underline{a}}}\underline{a}$$

$$= \frac{\underline{a}\cdot\underline{u}}{\underline{a}\cdot\underline{a}}\underline{a} = \text{proj}_{\underline{a}}\underline{u}$$

Bonus Question. *(2 marks)* What kind of geometrical object is $r(t) = (t^3, t^3, t^3)$ where $t \in \mathbb{R}$.