

Question 1. If the volume of the parallelepiped determined by the vectors $2\mathbf{u} + 3\mathbf{v}$, $3\mathbf{v} - \mathbf{w}$, and $\mathbf{u} - \mathbf{v}$ is equal to 20.

a. (5 marks) Find all the values of the scalar triple product $\mathbf{w} \cdot (\mathbf{v} \times \mathbf{u})$.

Question 2. (5 marks) Show that the lines $(L_1) : \begin{cases} x = 3 + t \\ y = 5 + t \\ z = 2 + t \end{cases}$, and $(L_2) : \begin{cases} x = -1 + 2s \\ y = 1 + s \\ z = s \end{cases}$, $t, s \in \mathbb{R}$ are skew lines, and find the distance between them.

Question 3. (4 marks) Let $W = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_{22} \mid 2a - b + 3c - d = 0 \right\}$. Determine whether W is a subspace of M_{22} .

Question 4. (3 marks) Show that the additive inverse of any vector in a vector space is unique. *Show every step, justify every step, and cite the axiom(s) used!!!*

Question 5. (2 marks) Determine whether $U = \{(r, 0, s) \mid r^2 + s^2 = 2 \text{ and } r, s \in \mathbb{R}\}$ is a vector space with the usual operations in \mathbb{R}^3 .