Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S6: Fall 2024: **Quiz 8** name: ________________________

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(3 marks)* Determine whether each statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof.

If $\mathbf{v}_1 = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$, then for any nonzero 2×2 matrix A, the set $\{A\mathbf{v}_1, A\mathbf{v}_2\}$ is linearly independent.

Question 2. *(5 marks)* Find a basis and dimension of $W = \operatorname{span}\left\{x^2 - x + 2,\ x - 1,\ -3x^2 + 3x - 6,\ 8x^2 + 7x + 1\right\}$.

Question 3. *(5 marks)*
Let $W = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_{22} \;\middle|\; 2a - b + 3c - d = 0 \right\}$. Find a basis and the dimension of W. Find the coordinates vector of $A = \begin{bmatrix} 2 & -1 \\ 1 & 8 \end{bmatrix}$ relative to the basis.

Question 4. *(5 marks)* Suppose that $U = \text{span}\{\mathbf{x}_1,\, \mathbf{x}_2,\, \ldots, \mathbf{x}_k\}$ where each $\mathbf{x}_i$ is in $\mathbb{R}^n$. If A is an $m \times n$ matrix and $A\mathbf{x}_i = \mathbf{0}$ for each i, show that $A\mathbf{y} = \mathbf{0}$ for every vector $\mathbf{y}$ in U.