Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S6: Fall 2024: **Quiz 5** name: Y. Lamontagne

Books, watches, notes or cell phones are **not** allowed. The **only** calculators allowed are the Sharp EL-531**. You **must** show all your work, the correct answer is worth 1 mark the remaining marks are given for the work.

Question 1. *(5 marks)* Let **u** and **v** be unit vectors, such that the angle between them is $\frac{2\pi}{3}$. Find $||5\mathbf{u} - 2\mathbf{v}||$.

$\|\underline{u}\| = 1$

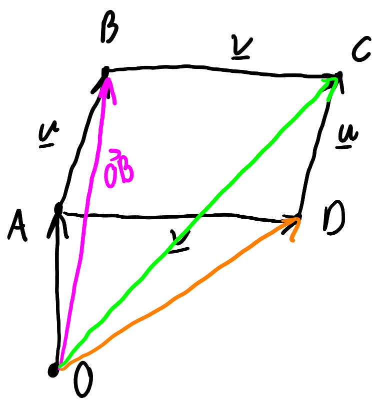
$\|\underline{v}\| = 1$

$$\begin{aligned}\underline{u}\cdot\underline{v} &= \|\underline{u}\|\|\underline{v}\|\cos\frac{2\pi}{3}\\ &= 1\cdot 1\left(-\frac{1}{2}\right)\\ &= -\frac{1}{2}\end{aligned}$$

$$\begin{aligned}\|5\underline{u}-2\underline{v}\|^2 &= (5\underline{u}-2\underline{v})\cdot(5\underline{u}-2\underline{v})\\ &= (5\underline{u})\cdot(5\underline{u}) + (5\underline{u})\cdot(-2\underline{v}) + (-2\underline{v})\cdot(5\underline{u}) + (-2\underline{v})\cdot(-2\underline{v})\\ &= 25\,\underline{u}\cdot\underline{u} - 10\,\underline{u}\cdot\underline{v} - 10\,\underline{v}\cdot\underline{u} + 4\,\underline{v}\cdot\underline{v}\\ &= 25\|\underline{u}\|^2 - 10\,\underline{u}\cdot\underline{v} - 10\,\underline{u}\cdot\underline{v} + 4\|\underline{v}\|^2\\ &= 25(1)^2 - 20\,\underline{u}\cdot\underline{v} + 4(1)^2\\ &= 29 - 20\left(\frac{-1}{2}\right)\\ &= 39\end{aligned}$$

$\therefore\ \|5\underline{u}-2\underline{v}\| = \sqrt{39}$

Question 2. *(5 marks)* Given a parallelogram with vertices A, B, C and D where $A(2,-2,4)$ and where the sides of the parallelogram are parallel to $\vec{u} = (1,2,3)$ and $\vec{v} = (-3,2,5)$. Find a set of vertices B, C and D.



$\vec{OB} = \vec{OA} + \underline{u} = (2,-2,4) + (1,2,3) = (3,0,7)$

$\therefore B(3,0,7)$

$\vec{OC} = \vec{OA} + \underline{u} + \underline{v} = (2,-2,4) + (1,2,3) + (-3,2,5) = (0,2,12)$

$\therefore C(0,2,12)$

$\vec{OD} = \vec{OA} + \underline{v} = (2,-2,4) + (-3,2,5) = (-1,0,9)$

$\therefore D(-1,0,9)$

note: not a unique solution

Question 3. *(5 marks)* Consider two 4×4 matrices A and B, with $\det(\mathrm{adj}(AB)) = -8$ and $\det(B) = 3$. Find the determinant of M, given

$$\det(\det(B)B^T MA^{-1}) = \det(5\mathrm{adj}(B)A^2).$$

$$(\det(AB))^{4-1} = -8$$
$$\det(AB) = -2$$
$$\det A \det B = -2$$
$$\det A = \frac{-2}{3}$$

$$(\det B)^4 \det(B^T M A^{-1}) = 5^4 \det(\mathrm{adj}(B)A^2)$$
$$3^4 \det(B^T)\det(M)\det A^{-1} = 5^4 \det(\mathrm{adj}(B))\det(A^2)$$
$$3^4 \det(B)\det(M)\frac{1}{\det A} = 5^4 (\det B)^{4-1}(\det A)^2$$
$$\det M = \frac{5^4}{3^4}\frac{\det B^3}{\det B}(\det A)^2 \det A$$
$$\det M = \frac{5^4}{3^4}(\det B)^2(\det A)^3$$
$$\det M = \frac{5^4}{3^{4-2}}\cancel{3^2}\left(\frac{-2}{3}\right)^3$$
$$\det M = -\frac{2^3 5^4}{3^5}$$

Question 4. *(3 marks)* Determine whether the following statement is true or false. If the statement is false provide a counterexample. If the statement is true provide a proof of the statement.

If A, B and C are $n \times n$ matrices such that $A\mathbf{x} = \mathbf{0}$ has infinitely many solutions then $(A^T C^{-1} + (BA)^T)\mathbf{x} = \mathbf{0}$ has infinitely many solutions.

True

By the equivalence thm since $A\underline{x} = \underline{0}$ has ∞-many solutions $\Rightarrow \det A = 0$

$$\det(A^T C^{-1} + (BA)^T) = \det(A^T C^{-1} + A^T B)$$
$$= \det(A^T(C^{-1} + B))$$
$$= \det A^T \det(C^{-1} + B)$$
$$= \det A \det(C^{-1} + B)$$
$$= 0 \cdot \det(C^{-1} + B) = 0$$

$\therefore$ by the equivalence thm. $(A^T C^{-1} + (BA)^T)\underline{x} = \underline{0}$ has infinitely many solutions.