Dawson College: Linear Algebra and Vector Geometry
201-SN4-RE-S03: Fall 2026: Test 1

Name: _____ Student ID: _____

Books, watches, notes or cell phones are not allowed. The only calculators allowed are the Sharp EL-531**. You must show all your work. The correct answer is worth 1 mark; the remaining marks are given for the work. If you need more space, use the back of the page and clearly indicate the question number.

This test is graded out of 56 marks.

Question 1. *(1 mark each)* Complete each sentence with **MUST**, **MIGHT**, or **CANNOT**.

a. Let A be a square matrix. If $A\mathbf{x} = A\mathbf{y}$ only when $\mathbf{x} = \mathbf{y}$, then A _____ be invertible.

b. A linear system with three equations and two variables _____ be inconsistent.

c. If A is a product of elementary matrices, then A _____ be singular.

d. If A and B are invertible $n \times n$ matrices, then $A + B$ _____ be invertible.

e. If A, B, and C are square matrices of the same size and $ABC^2 = I$, then C _____ be invertible.

Question 2. *(3 marks each)* Determine whether each statement is true or false. If a statement is false, provide a counterexample. If it is true, provide a proof.

a. Consider a system of linear equations with augmented matrix A. If there is more than one solution, then A has a row of zeros.

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Question 2 continued.

b. If A^2 is a symmetric matrix, then A is a symmetric matrix.

c. If A and B are square matrices of the same size, then $A^TB - B^TA$ is skew-symmetric.

Question 3. Given that X, A, B, and C are $n \times n$ matrices, solve the following equation for X. Assume any necessary matrices to be invertible. *(5 marks)*

$$2X^T + A = (XB - C)^T$$

Question 4. *(6 marks)*

Let

$$A = \begin{bmatrix} 1 & 0 & 2 \\ -1 & k & 6 \\ 0 & 2 & 4k \end{bmatrix} \qquad \text{and} \qquad \mathbf{b} = \begin{bmatrix} 1 \\ 3h \\ 2 \end{bmatrix}.$$

Find the values of h and k, if possible, for which the equation $A\mathbf{x} = \mathbf{b}$ has:

a. a unique solution;
b. infinitely many solutions;
c. no solution.

Question 5.

a. Use the inversion algorithm to find B^{-1}, where *(5 marks)*

$$B = \begin{bmatrix} 2 & 3 & 0 \\ 4 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

b. Using the inverse found in part (a), solve the linear system

$$\begin{cases} 2x + 3y = -1, \\ 4x + 5y = -1, \\ z = 2. \end{cases}$$

(2 marks)

Question 6. Let

$$A = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix}, \qquad B = \begin{bmatrix} 2d & b+2d & -3c & a+3c \\ 2h & f+2h & -3g & e+3g \\ 2l & j+2l & -3k & i+3k \\ 2p & n+2p & -3o & m+3o \end{bmatrix}.$$

Given that $\det(A) = 5$, compute $\det(B)$. Show your work and explain how the changes from A to B affect the determinant. *(5 marks)*

Question 7. Compute the following determinant. Show all your work. *(5 marks)*

$$\begin{vmatrix} 1 & 2 & 2 & 0 \\ 2 & 1 & -1 & 0 \\ 3 & 0 & 1 & 2 \\ 1 & 3 & 2 & -2 \end{vmatrix}.$$

Question 8. Find all matrices A, if any, such that *(5 marks)*

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} A = I_2.$$

Question 9. Find two elementary matrices E_1 and E_2 that satisfy *(4 marks)*

$$E_2E_1\begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 5\\ -2 & -2 & -11\end{bmatrix} = \begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 5\\ 0 & 0 & 1\end{bmatrix}.$$

Question 10. Let $A\mathbf{x} = \mathbf{0}$ be a homogeneous system of n linear equations in n unknowns, and let Q be an invertible $n \times n$ matrix. Using the Equivalence Theorem, prove that $A\mathbf{x} = \mathbf{0}$ has only the trivial solution if and only if $(QA)\mathbf{x} = \mathbf{0}$ has only the trivial solution. *(5 marks)*