Dawson College: Linear Algebra and Vector Geometry
201-SN4-RE-S04: Fall 2026: Test 1
Version A

Name: _____ Student ID: _____

Books, watches, notes or cell phones are not allowed. The only calculators allowed are the Sharp EL-531**. You must show all your work. The correct answer is worth 1 mark; the remaining marks are given for the work. If you need more space, use the back of the page and clearly indicate the question number.

This test is graded out of 56 marks.

Question 1. *(1 mark each)* Complete each sentence with **MUST**, **MIGHT**, or **CANNOT**.

a. If A is upper triangular, then A^T _____ be upper triangular.

b. A linear system with fewer equations than variables _____ have exactly one solution.

c. If E_1 and E_2 are elementary matrices of the same size, then E_1E_2 _____ be an elementary matrix.

d. If A is invertible, then A^2 _____ be invertible.

e. If A and B are symmetric matrices of the same size, then AB _____ be symmetric.

Question 2. *(3 marks each)* Determine whether each statement is true or false. If a statement is false, provide a counterexample. If it is true, provide a proof.

a. If two square matrices have the same reduced row echelon form, then they have the same determinant.

Continued on the next page

Question 2 continued.

b. If A is a square matrix and $A^2 = I$, then $A = I$ or $A = -I$.

c. If A is skew-symmetric and B is a square matrix of the same size, then $B^T AB$ is skew-symmetric.

Question 3. Given that X, A, B, and C are $n \times n$ matrices, solve the following equation for X. Assume any necessary matrices to be invertible. *(5 marks)*

$$3X^T + B = (CX - A)^T$$

Question 4. *(6 marks)*

Let

$$A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & k & 3 \\ 0 & 2 & 2k \end{bmatrix} \qquad \text{and} \qquad \mathbf{b} = \begin{bmatrix} 1 \\ 2h \\ 2 \end{bmatrix}.$$

Find the values of h and k, if possible, for which the equation $A\mathbf{x} = \mathbf{b}$ has:

a. a unique solution;
b. infinitely many solutions;
c. no solution.

Question 5.

a. Use the inversion algorithm to find B^{-1}, where *(5 marks)*

$$B = \begin{bmatrix} 3 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}.$$

b. Using the inverse found in part (a), solve the linear system

$$\begin{cases} 3x + y = 2, \\ 2x + y = 1, \\ 2z = 4. \end{cases}$$

(2 marks)

Question 6. Let

$$A = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix}, \qquad B = \begin{bmatrix} -2c & 3b & a-2c & d-3b \\ -2g & 3f & e-2g & h-3f \\ -2k & 3j & i-2k & l-3j \\ -2o & 3n & m-2o & p-3n \end{bmatrix}.$$

Given that $\det(A) = -4$, compute $\det(B)$. Show your work and explain how the changes from A to B affect the determinant. *(5 marks)*

Question 7. Compute the following determinant. Show all your work. *(5 marks)*

$$\begin{vmatrix} 2 & 0 & 1 & 3 \\ 1 & 0 & 2 & -1 \\ 3 & 2 & 0 & 1 \\ 1 & -2 & 3 & 2 \end{vmatrix}.$$

Question 8. Find all matrices X, if any, such that *(5 marks)*

$$X \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 1 & 1 \end{bmatrix} = I_2.$$

Question 9. Find two elementary matrices E_1 and E_2 that satisfy the following equation. *(4 marks)*

$$E_2E_1\begin{bmatrix}-2 & -3 & 0\\ 1 & 0 & 2\\ 0 & 1 & -1\end{bmatrix} = \begin{bmatrix}0 & 0 & 1\\ 1 & 0 & 2\\ 0 & 1 & -1\end{bmatrix}.$$

Question 10. Let A and B be $m \times n$ matrices. Prove that A and B are row equivalent if and only if they have the same reduced row echelon form. *(5 marks)*