Dawson College: Linear Algebra and Vector Geometry
201-SN4-RE-S04: Fall 2026: Test 1
Version B

Name: _____ Student ID: _____

Books, watches, notes or cell phones are not allowed. The only calculators allowed are the Sharp EL-531**. You must show all your work. The correct answer is worth 1 mark; the remaining marks are given for the work. If you need more space, use the back of the page and clearly indicate the question number.

This test is graded out of 56 marks.

Question 1. *(1 mark each)* Complete each sentence with **MUST**, **MIGHT**, or **CANNOT**.

a. If a linear system has two distinct solutions, then it _____ have infinitely many solutions.

b. A linear system whose augmented matrix has a leading entry in its last column _____ be consistent.

c. If $A + B$ is invertible, then A _____ be invertible.

d. If A is a square matrix and A^2 is invertible, then A _____ be invertible.

e. A square matrix all of whose entries are nonzero _____ have determinant zero.

Question 2. *(3 marks each)* Determine whether each statement is true or false. If a statement is false, provide a counterexample. If it is true, provide a proof.

a. If A is an $n \times n$ matrix and $A\mathbf{x} = \mathbf{b}$ is inconsistent for one vector $\mathbf{b}$, then it is inconsistent for every nonzero vector $\mathbf{b}$.

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Question 2 continued.

b. If A is a square matrix and $A^2 = A$, then $A = 0$ or $A = I$.

c. If A and B are symmetric matrices of the same size, then $AB - BA$ is skew-symmetric.

Question 3. Given that X, A, B, and C are $n \times n$ matrices, solve the following equation for X. Assume any necessary matrices to be invertible. *(5 marks)*

$$(XB)^T - 2X^T = CA^T$$

Question 4. *(6 marks)*

Let

$$A = \begin{bmatrix} 1 & 0 & 3 \\ -1 & k & 1 \\ 0 & 2 & 2k \end{bmatrix} \qquad \text{and} \qquad \mathbf{b} = \begin{bmatrix} 2 \\ 4h \\ 6 \end{bmatrix}.$$

Find the values of h and k, if possible, for which the equation $A\mathbf{x} = \mathbf{b}$ has:

a. a unique solution;
b. infinitely many solutions;
c. no solution.

Question 5.

a. Use the inversion algorithm to find B^{-1}, where *(5 marks)*

$$B = \begin{bmatrix} 2 & 1 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

b. Using the inverse found in part (a), solve the linear system

$$\begin{cases} 2x + y = 3, \\ 3x + 2y = 4, \\ 3z = 3. \end{cases}$$

(2 marks)

Question 6. Let

$$A = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix}, \qquad B = \begin{bmatrix} 3a & -2d & c+3a & b+2d \\ 3e & -2h & g+3e & f+2h \\ 3i & -2l & k+3i & j+2l \\ 3m & -2p & o+3m & n+2p \end{bmatrix}.$$

Given that $\det(A) = 7$, compute $\det(B)$. Show your work and explain how the changes from A to B affect the determinant. *(5 marks)*

Question 7. Compute the following determinant. Show all your work. *(5 marks)*

$$\begin{vmatrix} 2 & 1 & 0 & 3 \\ 1 & 2 & 0 & -1 \\ 3 & 0 & 3 & 1 \\ 1 & 3 & -3 & 2 \end{vmatrix}.$$

Question 8. Find all matrices X, if any, such that *(5 marks)*

$$X \begin{bmatrix} 1 & 0 \\ 3 & 1 \\ 2 & 1 \end{bmatrix} = I_2.$$

Question 9. Find two elementary matrices E_1 and E_2 that satisfy the following equation. *(4 marks)*

$$E_2E_1\begin{bmatrix}-3 & -2 & 2\\ 1 & 0 & 1\\ 0 & 1 & -2\end{bmatrix} = \begin{bmatrix}0 & 0 & 1\\ 1 & 0 & 1\\ 0 & 1 & -2\end{bmatrix}.$$

Question 10. Let A and B be $m \times n$ matrices. Prove that A and B are row equivalent if and only if there exists an invertible $m \times m$ matrix P such that $B = PA$. *(5 marks)*