Supplementary Problems

Linear Algebra and Vector Geometry for Social Science – 201-MA3-DW

4 Lines

4.1 [GHC] Write the vector and parametric equations of the lines described.

a. Passes through $P = (2, -4, 1)$, parallel to $\mathbf{d} = (9, 2, 5)$.

b. Passes through $P = (6, 1, 7)$, parallel to $\mathbf{d} = (-3, 2, 5)$.

c. Passes through $P = (2, 1, 5)$ and $Q = (7, -2, 4)$.

d. Passes through $P = (1, -2, 3)$ and $Q = (5, 5, 5)$.

e. Passes through $P = (0, 1, 2)$ and orthogonal to both $\mathbf{d}_1 = (2, -1, 7)$ and $\mathbf{d}_2 = (7, 1, 3)$.

f. Passes through $P = (5, 1, 9)$ and orthogonal to both $\mathbf{d}_1 = (1, 0, 1)$ and $\mathbf{d}_2 = (2, 0, 3)$.

g. Passes through the point of intersection of both lines and is orthogonal to both: $\mathbf{x} = (2, 1, 1) + t(5, 1, -2)$ and $\mathbf{x} = (-2, -1, 2) + t(3, 1, -1)$.

h. Passes through the point of intersection of both lines and is orthogonal to both: $\mathbf{x} = (t, -2 + 2t, 1 + t)$ and $\mathbf{x} = (2 + t, 2 - t, 3 + 2t)$.

i. Passes through $P = (1, 1)$, parallel to $\mathbf{d} = (2, 3)$.

j. Passes through $P = (-2, 5)$, parallel to $\mathbf{d} = (0, 1)$.

4.2

a. Find the point of intersection of the lines $\mathbf{x} = (2 + t, 5 + 4t, 3 + 2t)$ and $\mathbf{x} = (5 - 2s, 3 - s, -1 + s)$, where $s, t \in \mathbb{R}$.

b. Given $A = (2, -5, 7)$, the line $\mathbf{x} = (2 + t, -5 - 2t, 7 + 3t)$, and the plane $x - 2y + 3z = 5$, determine whether A lies on the line and whether A lies on the plane.

4.3 Find parametric equations for the line through $A = (1, 2, -5)$ that is perpendicular to the plane $x + 3y + 2z - 11 = 0$.

5 Planes

5.1 [GHC] Give the equation of the described plane in standard and general forms.

a. Passes through $(2, 3, 4)$ and has normal vector $\mathbf{n} = (3, -1, 7)$.

b. Passes through $(1, 3, 5)$ and has normal vector $\mathbf{n} = (0, 2, 4)$.

c. Passes through $(1, 2, 3)$, $(3, -1, 4)$ and $(1, 0, 1)$.

d. Passes through $(5, 3, 8)$, $(6, 4, 9)$ and $(3, 3, 3)$.

e. Contains the intersecting lines $\mathbf{x} = (2, 1, 2) + t(1, 2, 3)$ and $\mathbf{x} = (2, 1, 2) + t(2, 5, 4)$.

f. Contains the intersecting lines $\mathbf{x} = (5, 0, 3) + t(-1, 1, 1)$ and $\mathbf{x} = (1, 4, 7) + t(3, 0, -3)$.

g. Contains the parallel lines $\mathbf{x} = (1, 1, 1) + t(1, 2, 3)$ and $\mathbf{x} = (1, 1, 2) + t(1, 2, 3)$.

h. Contains the parallel lines $\mathbf{x} = (1, 1, 1) + t(4, 1, 3)$ and $\mathbf{x} = (2, 2, 2) + t(4, 1, 3)$.

i. Contains the point $(2, -6, 1)$ and the line $(x, y, z) = (2 + 5t, 2 + 2t, -1 + 2t)$.

j. Contains the point $(5, 7, 3)$ and the line $(x, y, z) = (t, t, t)$.

k. Contains the point $(5, 7, 3)$ and is orthogonal to the line $\mathbf{x} = (4, 5, 6) + t(1, 1, 1)$.

l. Contains the point $(4, 1, 1)$ and is orthogonal to the line $(x, y, z) = (4 + 4t, 1 + t, 1 + t)$.

5.2 [SM] Determine the scalar equation of the plane with the given vector equation.

a. $\mathbf{x} = (1, 4, 7) + s(2, 3, -1) + t(4, 1, 0),\ s, t \in \mathbb{R}$.

b. $\mathbf{x} = (2, 3, -1) + s(1, 1, 0) + t(-2, 1, 2),\ s, t \in \mathbb{R}$.

c. $\mathbf{x} = (1, -1, 3) + s(2, -2, 1) + t(0, 3, 1),\ s, t \in \mathbb{R}$.

5.3 [GHC] Find the point of intersection between the line and the plane.

a. Line: $(1, 2, 3) + t(3, 5, -1)$; plane: $3x - 2y - z = 4$.

b. Line: $(1, 2, 3) + t(3, 5, -1)$; plane: $3x - 2y - z = -4$.

c. Line: $(5, 1, -1) + t(2, 2, 1)$; plane: $5x - y - z = -3$.

d. Line: $(4, 1, 0) + t(1, 0, -1)$; plane: $3x + y - 2z = 8$.

5.4 [SM] Given the plane $2x_1 - x_2 + 3x_3 = 5$, for each line determine if it is parallel to the plane, orthogonal to the plane, or neither. If neither, determine the angle between the line's direction vector and the plane's normal vector.

a. $\mathbf{x} = (3, 0, 4) + t(-1, 1, 1),\ t \in \mathbb{R}$.

b. $\mathbf{x} = (1, 1, 2) + t(-2, 1, -3),\ t \in \mathbb{R}$.

c. $\mathbf{x} = (3, 0, 0) + t(1, 1, 2),\ t \in \mathbb{R}$.

d. $\mathbf{x} = (-1, -1, 2) + t(4, -2, 6),\ t \in \mathbb{R}$.

e. $\mathbf{x} = t(0, 3, 1),\ t \in \mathbb{R}$.

5.9 [SM] Determine a vector equation of the line of intersection of the given planes.

a. $x + 3y - z = 5$ and $2x - 5y + z = 7$.

b. $2x - 3z = 7$ and $y + 2z = 4$.

5.12(a) [BS] Given the lines $L_1 : (x, y, z) = (1, 3, 0) + t(4, 3, 1)$ and $L_2 : (x, y, z) = (1, 2, 3) + t(8, 6, 2)$, show that the lines lie in the same plane and find the general equation of this plane.

5.18 [YL] Find an equation of the plane that passes through the points $A = (1, 2, 3)$ and $B = (2, 3, 5)$ and is perpendicular to the plane $x + y = 0$.

5.19

a. Find an equation for the plane through $(2, 1, 3)$ that is perpendicular to both planes $x - y - z = 1$ and $2x - y + z = 2$.

b. Given $L_1 : x = 2 - t,\ y = 1 + t,\ z = 2$ and $L_2 : x = 2 + s,\ y = s,\ z = 2 + 2s$, find parametric equations for the line through $(0, -1, 2)$ that is perpendicular to both L_1 and L_2.

Answers

4.1

a. $\mathbf{x} = (2,-4,1)+t(9,2,5)$; $(x,y,z) = (2+9t,-4+2t,1+5t)$.

b. $\mathbf{x} = (6,1,7)+t(-3,2,5)$; $(x,y,z) = (6-3t,1+2t,7+5t)$.

c. $\mathbf{x} = (2,1,5)+t(5,-3,-1)$; $(x,y,z) = (2+5t,1-3t,5-t)$.

d. $\mathbf{x} = (1,-2,3)+t(4,7,2)$; $(x,y,z) = (1+4t,-2+7t,3+2t)$.

e. $\mathbf{x} = (0,1,2)+t(-10,43,9)$; $(x,y,z) = (-10t,1+43t,2+9t)$.

f. $\mathbf{x} = (5,1,9)+t(0,1,0)$; $(x,y,z) = (5,1+t,9)$.

g. The lines meet at $(7,2,-1)$. A direction vector is $(1,-1,2)$, so $\mathbf{x} = (7,2,-1)+t(1,-1,2)$.

h. The lines meet at $(2,2,3)$. A direction vector is $(5,-1,-3)$, so $\mathbf{x} = (2,2,3)+t(5,-1,-3)$.

i. $\mathbf{x} = (1,1)+t(2,3)$; $(x,y) = (1+2t,1+3t)$.

j. $\mathbf{x} = (-2,5)+t(0,1)$; $(x,y) = (-2,5+t)$.

4.2

a. $(1,1,1)$.

b. A lies on the line (at $t=0$), but not on the plane.

4.3 The plane normal is $(1,3,2)$, so $\mathbf{x} = (1,2,-5)+t(1,3,2)$; equivalently, $x = 1+t$, $y = 2+3t$, $z = -5+2t$.

5.1

a. $3(x-2)-(y-3)+7(z-4)=0$; $3x-y+7z-31=0$.

b. $2(y-3)+4(z-5)=0$; $y+2z-13=0$.

c. $2(x-1)+(y-2)-(z-3)=0$; $2x+y-z-1=0$.

d. $-5(x-5)+3(y-3)+2(z-8)=0$; $-5x+3y+2z=0$.

e. $-7(x-2)+2(y-1)+(z-2)=0$; $-7x+2y+z+10=0$.

f. $(x-1)+(z-4)=0$; $x+z-8=0$.

g. $2(x-1)-(y-1)=0$; $2x-y-1=0$.

h. $-2(x-1)-(y-1)+3(z-1)=0$; $-2x-y+3z=0$.

i. $-2(x-2)+(y-2)+4(z+1)=0$; $-2x+y+4z+6=0$.

j. $-2(x-5)+(y-7)+(z-3)=0$; $-2x+y+z=0$.

k. $(x-5)+(y-7)+(z-3)=0$; $x+y+z-15=0$.

l. $4(x-4)+(y-1)+(z-1)=0$; $4x+y+z-18=0$.

5.2

a. $x-4y-10z=-85$.

b. $2x-2y+3z=-5$.

c. $-5x-2y+6z=15$.

5.3

a. No intersection; the line is parallel to the plane.

b. The line lies in the plane, so there are infinitely many intersection points.

c. $\left(\frac{27}{7},-\frac{1}{7},-\frac{11}{7}\right)$.

d. $(3,1,1)$.

5.4

a. Parallel to the plane.

b. Orthogonal to the plane.

c. Neither; angle with the normal is $\cos^{-1}\left(\frac{7}{\sqrt{84}}\right) = \cos^{-1}\left(\frac{7}{2\sqrt{21}}\right)$.

d. Orthogonal to the plane.

e. Parallel to the plane.

5.9

a. $\mathbf{x} = (4,0,-1)+t(2,3,11)$.

b. $\mathbf{x} = (5,2,1)+t(3,-4,2)$.

5.12(a) The direction vectors are parallel, and the point difference $(0,-1,3)$ is not parallel to them, so the lines are distinct and coplanar. The plane is $5x-6y-2z=-13$.

5.18 $y-x=1$.

5.19

a. $-2x-3y+z+4=0$.

b. $x=t$, $y=-1+t$, $z=2-t$.