DAWSON COLLEGE
MATHEMATICS DEPARTMENT
FINAL EXAMINATION
CALCULUS II
201-NYB-05 (Science)
December 13, 2005   14:00-17:00

Student Name: _____

Student Number: _____

Circle the name of your instructor:

    D. DUBROVSKY   *T. FOX*   *B. SZCZEPARA*

TIME: 3 hours

INSTRUCTIONS:

- Print your name and student I.D. number in the space provided above.

- All questions are to be answered directly on the examination in the provided space.

- Translation and regular dictionaries are permitted.

- Only small, noiseless and non-programmable calculators are permitted.

- This examination consists of 11 questions divided into a total of 20 parts, each part worth 5 marks.

  Please ensure that you have a complete examination before starting.

- This exam must be returned intact.

1. Evaluate

    (a) $\displaystyle\int \frac{\ell n\, x}{x}\, dx$

    (b) $\displaystyle\int xe^{3x}\, dx$

    (c) $\displaystyle\int \frac{4x+5}{(x-1)(x+2)^2}\, dx$

    (d) $\displaystyle\int \frac{x^2}{\sqrt{1-x^2}}\, dx$

2. Find the area of the region enclosed by the graphs of $y = x^2 - 2x$ and $y = 2 - x$.

3. Find the volume of the solid obtained when the region bounded by the graphs of $y = x$ and $y = x^2$ is rotated (a) about the x-axis (b) about the y-axis.

4. A tank 10 m high is located with its base at ground level. Its shape is formed by revolving the parabola $y = \frac{x^2}{4}$ about the y-axis. It has been filled to a depth of 6 m with oil of mass density ρ. Find the work required to pump all the oil out to a reservoir 3 meters above the top of the tank.

5. Find the length of the graph of $f(x) = \frac{4}{5} x^{5/4}$ for $0 \le x \le 1$.

6. Find the limit or show it does not exist.

    (a) $\displaystyle\lim_{x\to 0^+} \frac{e^{2x} - 2x - 1}{\cos x - 1}$

    (b) $\displaystyle\lim_{x\to\infty}\left(1 - \frac{1}{x^2}\right)^{x^2}$

7. Evaluate the integral or show it diverges.

    (a) $\displaystyle\int_0^\infty \frac{dx}{9 + x^2}$

    (b) $\displaystyle\int_0^1 \frac{dx}{\sqrt{1-x}}$

8. Find the sum of the series or show it diverges.

    (a) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n - 5^n}{7^n}$

    (b) $\displaystyle\sum_{n=1}^{\infty} \frac{2n-3}{3n-2}$

9. Determine whether each of the following series converges or diverges. Justify your answer by using an appropriate test.

    (a) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{(n+1)!}$

    (b) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ell n\, n}}$

    (c) $\displaystyle\sum_{n=1}^{\infty} \frac{2n-3}{3n^2 - 2}$

10. Find the McLaurin polynomial of degree 5 for $f(x) = \ln(1+x)$.

11. What does the Fundamental Theorem of Calculus say? Why is it important?