

Name: _____
Student ID: _____

Test 2

This test is graded out of 45 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (5 marks) Find the average of the function $f(x) = x \sin x^2$ over the interval $[-\sqrt{\pi}, \sqrt{\pi}]$.

Question 2. (5 marks) Evaluate the following definite integral:

$$\int_0^2 \frac{1}{x^2 - 2x + 2} dx$$

Question 3. (2 marks for sketch and 3 marks for area) Sketch the graph of the two following algebraic functions $f(x) = -x$ and $g(x) = -x^2 + 2x$ and find the area bounded by the two functions.

Question 4. (5 marks) Find the volume of the solid of revolution generated by the bounded region of the functions $f(x) = x^2 - 4x + 3$ and $g(x) = 3 + 4x - x^2$ revolved about the y -axis.

Question 5. (5 marks) Find the volume of the solid of revolution generated by the bounded region of the functions $f(x) = \sqrt{x}$ and $g(x) = 2 - x$ and the relations $x = 1$ and $x = 2$ revolved about the x -axis.

Question 6. (5 marks) Find the arc length of the graph of the function $f(x) = \frac{3}{2}x^{2/3} + 5$ over the interval $[0, 8]$.

Question 7. (5 marks) Use the Trapezoidal Rule with $n = 4$ to approximate the value of the definite integral and compare your answer to the exact value of the definite integral. (i.e. calculate the definite integral using the Fundamental Theorem of Calculus.)

$$\int_0^2 \frac{x^2}{\sqrt{1+x^3}} dx$$

Question 8. (5 marks) A hemispherical tank with its round part pointing downwards is filled by a pump which is located 5m below the tank. If the fluid that has a density of $\rho = 2000 \frac{\text{kg}}{\text{m}^3}$ and the tank is 4m across at the top, how much work is required to fill the tank?

Question 9. (5 marks) Find the 'c' value(s) guaranteed by the Mean Value Theorem for Integrals for the function $f(x) = \sec x$ over $[-\frac{\pi}{4}, \frac{\pi}{4}]$.

Bonus Question. (5 marks)

Prove one of the following statement:

- If $f(x)$ is an odd function then

$$\int_{-a}^a f(x) dx = 0$$

- The Fundamental Theorem of Calculus.