

Quiz 2

This quiz is graded out of 10 marks. No books, graphing calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (5 marks) Use the Simpson's Rule with $n = 4$ to approximate the value of the definite integral and compare your answer to the actual value of the integral. (i.e. Also calculate the definite integral using the Fundamental Theorem of Calculus.)

$$\int_1^2 \frac{1}{(x+1)^2} dx$$

$$\Delta x = \frac{b-a}{n} = \frac{2-1}{4} = \frac{1}{4}$$

$$x_0 = 1, x_1 = a + \Delta x = \frac{5}{4}, x_2 = a + 2\Delta x = \frac{6}{4}, x_3 = a + 3\Delta x = \frac{7}{4}, x_4 = 2$$

$$\approx \frac{b-a}{3n} \left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4) \right]$$

$$= \frac{2-1}{3(4)} \left[\frac{1}{(1+1)^2} + 4 \frac{1}{(\frac{5}{4}+1)^2} + 2 \frac{1}{(\frac{6}{4}+1)^2} + 4 \frac{1}{(\frac{7}{4}+1)^2} + \frac{1}{(2+1)^2} \right]$$

$$= \frac{1}{12} \left[\frac{1}{4} + \frac{4}{(\frac{9}{4})^2} + \frac{2}{(\frac{5}{2})^2} + \frac{4}{(\frac{11}{4})^2} + \frac{1}{9} \right]$$

$$\approx 0.1667$$

$$\int_1^2 \frac{1}{(x+1)^2} dx$$

$$u = x+1$$

$$du = dx$$

$$u(1) = 1+1 = 2$$

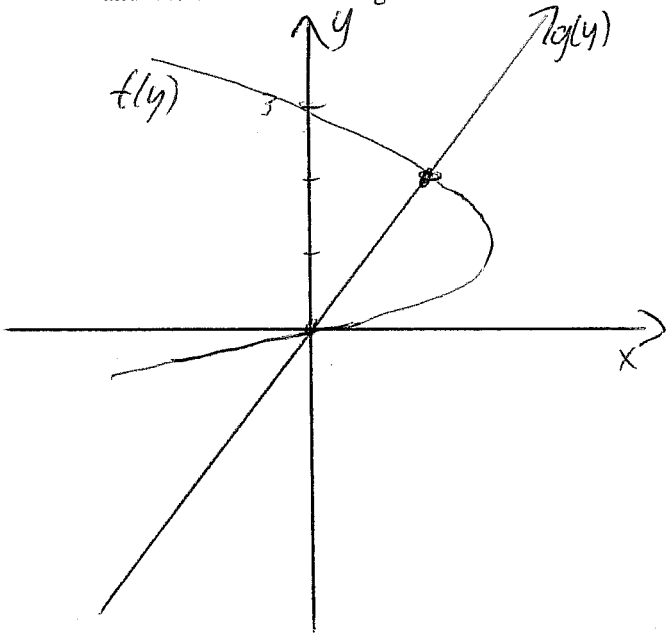
$$u(2) = 2+1 = 3$$

$$= \int_2^3 \frac{1}{u^2} du$$

$$= \left[\frac{-1}{u} \right]_2^3$$

$$= \frac{-1}{3} + \frac{1}{2} = \frac{1}{6}$$

Question 2. (5 marks) Sketch the region bounded by the graphs of the algebraic functions $f(y) = y(3-y)$ and $g(y) = y$ and find the area of the region.



Lets find the intersection
of $f(y)$ and $g(x)$

$$\begin{aligned} f(y) &= g(y) \\ y(3-y) &= y \\ 3y - y^2 &= y \\ 2y - y^2 &= 0 \\ y(2-y) &= 0 \end{aligned}$$

\therefore intersection at $y=0$
and $y=2$

$$\begin{aligned} \text{Area} &= \int_0^2 f(y) - g(y) dy \\ &= \int_0^2 3y - y^2 - y dy \\ &= \int_0^2 2y - y^2 dy \\ &= \left[y^2 - \frac{y^3}{3} \right]_0^2 \\ &= 2^2 - \frac{2^3}{3} \\ &= 4 - \frac{8}{3} \\ &= \frac{4}{3} \end{aligned}$$