Dawson College: **Calculus II**: 201-NYB-05-S2: Summer 2008

Name: ____________________
Student ID: ____________________

Quiz 4

This quiz is graded out of 12 marks. No books, graphing calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(4 marks each)* Determine if the series converges or diverges, justify by applying the correct test. If the series converges, find the sum.

1. $$\sum_{n=1}^{\infty} \frac{3n^2}{n^3+1}$$

2. $$\sum_{n=1}^{\infty} \frac{n^4+1}{n^4+n^2+1}$$

3. $$\sum_{n=1}^{\infty} \left[\frac{1}{n} - \frac{1}{n+1}\right]$$

(2) Let's try the nth term divergence test

$$\lim_{n\to\infty} \frac{n^4+1}{n^4+n^2+1} = 1$$

since the limit is not equal to 0 then $\sum_{n=1}^{\infty} \frac{n^4+1}{n^4+n^2+1}$ diverges.

(3) Appears to be a telescoping series, therefore we will take the partial sum.

$$S_n = a_1 + a_2 + a_3 + \ldots + a_{n-2} + a_{n-1} + a_n$$

$$= \left[\frac{1}{1} - \cancel{\frac{1}{1+1}}\right] + \left[\cancel{\frac{1}{2}} - \cancel{\frac{1}{2+1}}\right] + \left[\cancel{\frac{1}{3}} - \frac{1}{3+1}\right] + \ldots +$$

$$\left[\frac{1}{n-2} - \cancel{\frac{1}{n-2+1}}\right] + \left[\cancel{\frac{1}{n-1}} - \cancel{\frac{1}{n-1+1}}\right] + \left[\cancel{\frac{1}{n}} - \frac{1}{n+1}\right]$$

$$= 1 - \frac{1}{n+1}$$

$$S = \lim_{n\to\infty} S_n$$

$$= \lim_{n\to\infty} 1 - \cancel{\frac{1}{n+1}}^{\,0}$$

$$= 1$$

① Can either use the limit comparison test or Integral test. Let's use the integral test. Lets verify the conditions in order to use the test.

Let $f(x) = \frac{3x^2}{x^3+1}$

positive for $x \geq 1$? yes

continuous for $x \geq 1$? yes

decreasing for $x > 1$?

$$f'(x) = \frac{(6x(x^3+1) - 3x^2(3x^2))}{(x^3+1)^2}$$

$$= \frac{1-3x^4}{(x^3+1)^2} < 0$$

$$\int_1^\infty \frac{3x^2}{x^3+1}dx = \lim_{b\to\infty} \int_1^b \frac{3x^2}{x^3+1}dx$$

$$= \lim_{b\to\infty} \Big[\ln|x^3+1|\Big]_1^b$$

$$= \lim_{b\to\infty} \ln|b^3+1| - \ln|2|$$

∞

$\therefore$ diverges

$\therefore$ the series diverges by the integral test