Name: _____
Student ID: _____

Quiz 4

This quiz is graded out of 12 marks. No books, graphing calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(4 marks each)* Determine if the series converges or diverges, justify by applying the correct test. If the series converges, find the sum.

1. $\sum_{n=1}^{\infty} \frac{3n^2}{1+9n^2}$

2. $\sum_{n=1}^{\infty} \frac{n+1}{\sqrt{n^3+n^2+1}}$

3. $\sum_{n=1}^{\infty} \left[\frac{1}{n+1} - \frac{1}{n+2}\right]$

① Let's try the n^{th} term divergence test

$\lim_{n\to\infty} \frac{3n^2}{1+9n^2} = \frac{1}{3}$

since the limit is not equal to 0 then $\sum_{n=1}^{\infty} \frac{3n^2}{1+9n^2}$ diverges.

② Let's choose $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^3}}$ as a test series,

$\sum_{n=1}^{\infty} \frac{n}{n^{3/2}} = \sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$ it is a p-series that diverges since $\frac{1}{2} \leq 1$.

$\lim_{n\to\infty} \frac{n+1}{\sqrt{n^3+n^2+1}} = \lim_{n\to\infty} \frac{n+1}{n} \cdot \frac{\sqrt{n^3}}{\sqrt{n^3+n^2+1}}$

$= \lim_{n\to\infty} \frac{n+1}{n} \lim_{n\to\infty} \sqrt{\frac{n^3}{n^3+n^2+1}} = \lim_{n\to\infty} 1 + \frac{1}{n}^{0} \lim_{n\to\infty} \sqrt{\frac{1}{\frac{n^3}{n^3}+\frac{n^2}{n^3}+\frac{1}{n^3}}}^{0}$

$= 1$

∴ the limit is finite and positive, the series behaves like the test series. It diverges by the limit comparison test.

(3) appears to be a telescoping series. Lets find its partial sum.

$$S_n = a_1 + a_2 + a_3 + \dots + a_{n-2} + a_{n-1} + a_n$$

$$= \left[\frac{1}{1+1} - \frac{1}{1+2}\right] + \left[\frac{1}{2+1} - \frac{1}{2+2}\right] + \left[\frac{1}{3+1} - \frac{1}{3+2}\right]$$

$$+ \dots +$$

$$\left[\frac{1}{n-2+1} - \frac{1}{n-2+2}\right] + \left[\frac{1}{n-1+1} - \frac{1}{n-1+2}\right] + \left[\frac{1}{n+1} - \frac{1}{n+2}\right]$$

$$= \frac{1}{2} - \frac{1}{n+2}$$

$$S = \lim_{n\to\infty} S_n$$

$$= \lim_{n\to\infty} \frac{1}{2} - \frac{1}{n+2} \quad \left(\frac{1}{n+2} \to 0\right)$$

$$= \frac{1}{2}$$