Dawson College: **Calculus II**: 201-NYB-05-S2: Summer 2008

Name: ____________________
Student ID: ____________________

Test 1

This test is graded out of 45 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Formula:

$\sum_{i=1}^{n} c = cn$ where c is a constant $\qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

Question 1. *(3 marks)* Integrate the following indefinite integral:

$$\int \frac{1}{x^{2/5}} + x^{2/5} + \tan x \, dx = \frac{5x^{3/5}}{3} + \frac{5x^{7/5}}{7} - \ln|\cos x| + C$$

Question 2. *(5 marks)* Evaluate the definite integral using first principles *(i.e. limit process)*:

$$\int_0^2 2x^2 + x \, dx \qquad \Delta x = \frac{b-a}{n} = \frac{2}{n}, \quad x_i = a + i\Delta x = \frac{2i}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(2\left(\frac{2i}{n}\right)^2 + \left(\frac{2i}{n}\right)\right)\frac{2}{n}$$

$$= \lim_{n\to\infty} \frac{2}{n} \sum_{i=1}^{n} \frac{8i^2}{n^2} + \frac{2i}{n}$$

$$= \lim_{n\to\infty} \frac{2}{n}\left[\sum_{i=1}^{n} \frac{8i^2}{n^2} + \sum_{i=1}^{n} \frac{2i}{n}\right]$$

$$= \lim_{n\to\infty} \frac{2}{n}\left[\frac{8}{n^2}\sum_{i=1}^{n} i^2 + \frac{2}{n}\sum_{i=1}^{n} i\right]$$

$$= \lim_{n\to\infty} \frac{2}{n}\left[\frac{8}{n^{\cancel{2}}} \frac{\cancel{n}(n+1)(2n+1)}{6} + \frac{\cancel{2}}{\cancel{n}} \frac{n(n+1)}{\cancel{2}}\right]$$

$$= \lim_{n\to\infty} \frac{16}{6}\left(\frac{2n^2+3n+1}{n^2}\right) + \frac{2n}{n} + \cancel{\frac{2}{n}}^{\,0}\Big]$$

$$= \lim_{n\to\infty}\left[\frac{16}{6}\left(\frac{2\cancel{n^2}}{\cancel{n^2}} + \cancel{\frac{3n}{n^2}}^{\,0} + \cancel{\frac{1}{n^2}}^{\,0}\right) + 2\right] = \frac{22}{3}$$

Question 3. *(5 marks)* Integrate the following indefinite integral:

$$\int \frac{e^{3x}}{e^{6x}+1}\,dx = \int \frac{e^{3x}}{(e^{3x})^2+1}\,dx$$

① $u = e^{3x}$

$du = 3e^{3x}\,dx$

② $\frac{du}{3e^{3x}} = dx$

$$\overset{①,②}{=} \int \frac{e^{3x}}{u^2+1}\left(\frac{du}{3\cancel{e^{3x}}}\right)$$

$$= \frac{1}{3}\int \frac{1}{u^2+1}\,du$$

$$= \frac{1}{3}\arctan u + C$$

$$\overset{①}{=} \frac{1}{3}\arctan e^{3x} + C$$

Question 4. *(5 marks)* Integrate the following indefinite integral:

$$\int \sqrt{\sec 3x}\sec 3x \tan 3x\,dx$$

① $u = \sec 3x$

$du = 3\sec 3x \tan 3x\,dx$

② $\frac{du}{3} = \sec 3x \tan 3x\,dx$

$$\overset{①,②}{=} \int \sqrt{u}\,\frac{du}{3}$$

$$= \frac{1}{3}\int \sqrt{u}\,du$$

$$= \frac{1}{3}\,\frac{2u^{3/2}}{3} + C$$

$$= \frac{2u^{3/2}}{9} + C$$

$$= \frac{2(\sec 3x)^{3/2}}{9} + C$$

Question 5. Given $\int_a^b f(x)\,dx = 3$, $\int_a^c g(x)\,dx = 3$ and $\int_b^c f(x)\,dx = 4$ evaluate the following definite integrals:

1. *(1 mark)*

$$\int_a^b 3f(x)\,dx = 3\int_a^b f(x)dx = 3(3) = 9$$

2. *(3 marks)*

$$\int_c^a f(x) - 2g(x)\,dx = \int_c^a f(x)dx - 2\int_c^a g(x)dx$$
$$= -\left[\int_a^b f(x)dx + \int_b^c f(x)dx\right] + 2\int_a^c g(x)dx$$
$$= -\left[3+4\right] + 2\left[3\right] = -1$$

Question 6. *(5 marks)* Evaluate the following definite integral:

$$\int_0^{\pi/4} \frac{\sec^2 x}{1+\tan x}\,dx \overset{①,②}{=} \int_1^2 \frac{\sec^2 x}{u}\,\frac{du}{\sec^2 x}$$

① $u = 1 + \tan x$

$du = \sec^2 x\,dx$

② $\frac{du}{\sec^2 x} = dx$

$u(0) = 1 + \tan(0)$
$= 1 + 0$
$= 1$

$u(\pi/4) = 1 + \tan \pi/4$
$= 1 + 1$
$= 2$

$$= \int_1^2 \frac{1}{u}\,du$$
$$= \Big[\ln|u|\Big]_1^2$$
$$= \ln 2 - \ln 1$$
$$= \ln 2$$

Question 7. *(3 marks)* Use the Second Fundamental Theorem of Calculus to find $F'(x)$.

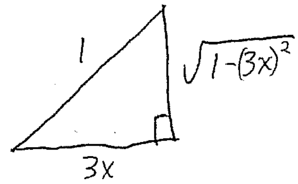
$$f(g(x)) = F(x) = \int_0^{\cos 3x} \arcsin y \, dy$$

Let $g(x) = \cos 3x \Rightarrow g'(x) = -\sin 3x \,(3)$

$f(x) = \int_0^{x} \arcsin y \, dy \Rightarrow f'(x) = \arcsin x$

(by the second fundamental theorem of calculus)

$$\frac{d}{dx}\left[f(g(x))\right] = f'(g(x))\,g'(x)$$
$$= \arcsin \cos 3x \cdot (-3 \sin 3x)$$
$$= \sqrt{1-9x^2}\,(-3 \sin 3x)$$
$$= -3\sqrt{1-9x^2}\,\sin 3x$$



Question 8. *(5 marks)* Integrate the following indefinite integral:

$$\int \frac{(\ln x)^2}{x}\,dx \overset{①②}{=} \int \frac{u^2 \cancel{x}\,du}{\cancel{x}}$$

① $u = \ln x$

$du = \frac{dx}{x}$

② $x\,du = dx$

$$= \int u^2\,du$$
$$= \frac{u^3}{3} + C$$
$$\overset{①}{=} \frac{(\ln x)^3}{3} + C$$

Question 9. *(5 marks)* Integrate the following indefinite integral:

$$\int \frac{e^{\sqrt{3x}}}{\sqrt{3x}}\,dx \overset{①,②}{=} \int \frac{e^{u}}{\sqrt{3x}} \frac{2\sqrt{3x}}{3}\,du$$

① $u = \sqrt{3x}$

$du = \frac{3\,dx}{2\sqrt{3x}}$

② $\frac{2\sqrt{3x}\,du}{3} = dx$

$$= \frac{2}{3}\int e^{u}\,du$$

$$= \frac{2}{3} e^{u} + C$$

$$\overset{①}{=} \frac{2}{3} e^{\sqrt{3x}} + C$$

Question 10. *(5 marks)* Evaluate the following definite integral:

$$\int_{-3}^{-2} \frac{x^4+x^3}{4x^5+5x^4+1}\,dx \overset{①,②}{=} \int_{-566}^{-47} \frac{\cancel{x^4+x^3}}{u} \left(\frac{du}{20\,(\cancel{x^4+x^3})}\right)$$

① $u = 4x^5 + 5x^4 + 1$

$du = 20x^4 + 20x^3\,dx$

② $dx = \frac{du}{20\,(x^4+x^3)}$

$u(-3) = 4(-3)^5 + 5(-3)^4 + 1$

$= -566$

$u(-2) = 4(-2)^5 + 5(-2)^4 + 1$

$= -47$

$$= \frac{1}{20}\int_{-566}^{-47} \frac{1}{u}\,du$$

$$= \frac{1}{20}\Big[\ln|u|\Big]_{-566}^{-47}$$

$$= \frac{1}{20}\Big[\ln|-47| - \ln|-566|\Big]$$

$$= \ln \sqrt[20]{\frac{47}{566}}$$

Bonus Question. *(3 marks)*
Integrate the following indefinite intergral:

$$\int \frac{1}{x(\operatorname{arcsec} x)(\ln \operatorname{arcsec} x)\sqrt{x^2-1}}\,dx \overset{①,②}{=} \int \frac{1}{u}\,du$$

① $u = \ln \operatorname{arcsec} x$

② $du = \frac{dx}{\operatorname{arcsec} x\,(x)\sqrt{x^2-1}}$

$$= \ln|u| + C$$

$$= \ln|\ln \operatorname{arcsec} x| + C$$