

Name: _____
Student ID: _____

Test 1

This test is graded out of 45 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Formula:

$$\sum_{i=1}^n c = cn \quad \text{where } c \text{ is a constant} \quad \sum_{i=1}^n i = \frac{n(n+1)}{2}$$
$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6} \quad \sum_{i=1}^n i^3 = \frac{n^2(n+1)^2}{4}$$

Question 1. (3 marks) Integrate the following indefinite integral:

$$\int \frac{1}{x^{2/7}} + x^{2/7} + \cot x \, dx$$

Question 2. (5 marks) Evaluate the definite integral using first principles (*i.e. limit process*):

$$\int_0^1 x^3 + x \, dx$$

Question 3. (5 marks) Integrate the following indefinite integral:

$$\int \frac{e^{2x}}{e^{4x} + 1} dx$$

Question 4. (5 marks) Integrate the following indefinite integral:

$$\int \sqrt{\csc 5x} \csc 5x \cot 5x dx$$

Question 5. Given $\int_a^b g(x) \, dx = 1$, $\int_a^c f(x) \, dx = 2$ and $\int_b^c g(x) \, dx = 3$ evaluate the following definite integrals:

1. (1 mark)

$$\int_a^c 3f(x) \, dx$$

2. (3 marks)

$$\int_c^a f(x) - 2g(x) \, dx$$

Question 6. (5 marks) Evaluate the following definite integral:

$$\int_0^{\pi/2} \frac{\sin \frac{x}{2}}{1 + \cos \frac{x}{2}} \, dx$$

Question 7. (3 marks) Use the Second Fundamental Theorem of Calculus to find $F'(x)$.

$$F(x) = \int_0^{\sin 5x} \frac{1}{\sqrt{1-y^2}} dy$$

Question 8. (5 marks) Integrate the following indefinite integral:

$$\int \frac{(\ln x)^3}{x} dx$$

Question 9. (5 marks) Integrate the following indefinite integral:

$$\int \frac{e^{\sqrt[3]{x}}}{\sqrt[3]{x^2}} dx$$

Question 10. (5 marks) Evaluate the following definite integral:

$$\int_{-3}^{-2} \frac{x^2 + x}{2x^3 + 3x^2 + 1} dx$$

Bonus Question. (3 marks)

Integrate the following indefinite integral:

$$\int \frac{1}{x(\operatorname{arcsec} x)(\ln \operatorname{arcsec} x)\sqrt{x^2 - 1}} dx$$