Dawson College: **Calculus II**: 201-NYB-05-S2: Summer 2008

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Test 1

This test is graded out of 45 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Formula:

$\sum_{i=1}^{n} c = cn$ where c is a constant $\quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \quad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

Question 1. *(3 marks)* Integrate the following indefinite integral:

$$\int \frac{1}{x^{2/7}} + x^{2/7} + \cot x \, dx = \frac{7X^{5/7}}{5} + \frac{7X^{9/7}}{9} + \ln|\sin x| + C$$

Question 2. *(5 marks)* Evaluate the definite integral using first principles *(i.e. limit process)*:

$$\int_0^1 x^3 + x\, dx \qquad \Delta X = \frac{b-a}{n} = \frac{1}{n}, \quad X_i = a + i\Delta X = \frac{i}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\left(\frac{i}{n}\right)^3 + \left(\frac{i}{n}\right)\right)\frac{1}{n}$$

$$= \lim_{n\to\infty} \frac{1}{n}\left[\sum_{i=1}^{n}\left(\frac{i}{n}\right)^3 + \sum_{i=1}^{n}\left(\frac{i}{n}\right)\right]$$

$$= \lim_{n\to\infty} \frac{1}{n}\left[\frac{1}{n^3}\sum_{i=1}^{n} i^3 + \frac{1}{n}\sum_{i=1}^{n} i\right]$$

$$= \lim_{n\to\infty} \frac{1}{n^{\cancel{4}2}}\left[\frac{\cancel{n^2}(n+1)^2}{4}\right] + \frac{1}{n^{\cancel{2}}}\left[\frac{\cancel{n}(n+1)}{2}\right]$$

$$= \lim_{n\to\infty}\left[\frac{n^2+2n+1}{n^2 4} + \frac{n+1}{n2}\right]$$

$$= \lim_{n\to\infty}\left[\frac{\cancel{n^2}}{4\cancel{n^2}} + \cancel{\frac{2n}{4n^2}}^{0} + \cancel{\frac{1}{4n^2}}^{0} + \frac{\cancel{n}}{2\cancel{n}} + \cancel{\frac{1}{2n}}^{0}\right]$$

$$= \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$$

Question 3. *(5 marks)* Integrate the following indefinite integral:

$$\int \frac{e^{2x}}{e^{4x}+1}\,dx = \int \frac{e^{2x}}{(e^{2x})^2+1}\,dx \overset{①②}{=} \int \frac{e^{2x}}{u^2+1}\,\frac{du}{2\cancel{e^{2x}}}$$

① $u = e^{2x}$

$du = 2e^{2x}\,dx$

② $\frac{du}{2e^{2x}} = dx$

$$= \frac{1}{2}\int \frac{du}{u^2+1}$$

$$= \frac{1}{2}\arctan u + C$$

$$\overset{①}{=} \frac{1}{2}\arctan e^{2x} + C$$

Question 4. *(5 marks)* Integrate the following indefinite integral:

$$\int \sqrt{\csc 5x}\,\csc 5x \cot 5x\,dx \overset{①,②}{=} \int \sqrt{u}\,\frac{du}{-5}$$

① $u = \csc 5x$

$du = -5\csc 5x \cot 5x\,dx$

② $\frac{du}{-5} = \csc 5x \cot 5x\,dx$

$$= \frac{1}{-5}\int \sqrt{u}\,du$$

$$= \frac{-1}{5}\,\frac{2u^{3/2}}{3} + C$$

$$= \frac{-2u^{3/2}}{15} + C$$

$$\overset{①}{=} \frac{-2(\csc 5x)^{3/2}}{15} + C$$

Question 5. Given $\int_a^b g(x)\,dx = 1$, $\int_a^c f(x)\,dx = 2$ and $\int_b^c g(x)\,dx = 3$ evaluate the following definite integrals:

1. *(1 mark)*

$$\int_a^c 3f(x)\,dx = 3\int_a^c f(x)dx = 3(2) = 6$$

2. *(3 marks)*

$$\int_c^a f(x) - 2g(x)\,dx = \int_c^a f(x)\,dx - 2\int_c^a g(x)\,dx$$

$$= -\int_a^c f(x)dx + 2\left[\int_a^b g(x)dx + \int_b^c g(x)dx\right]$$

$$= -2 + 2[1+3] = 6$$

Question 6. *(5 marks)* Evaluate the following definite integral:

$$\int_0^{\pi/2} \frac{\sin\frac{x}{2}}{1+\cos\frac{x}{2}}\,dx$$

① $u = 1 + \cos\frac{x}{2}$

$du = -\sin\frac{x}{2}\left(\frac{1}{2}\right)dx$

② $\frac{-2\,du}{\sin x} = dx$

$u(0) = 1 + \cos 0$
$= 1 + 1$
$= 2$

$u(\pi/2) = 1 + \cos\pi/4$
$= 1 + \frac{1}{\sqrt{2}}$
$= \frac{\sqrt{2}+1}{\sqrt{2}}$

$$\underset{=}{①,②} \int_2^{\frac{\sqrt{2}+1}{\sqrt{2}}} \frac{\cancel{\sin x}}{u}\,\frac{-2\,du}{\cancel{\sin x}}$$

$$= -2\int_2^{\frac{\sqrt{2}+1}{\sqrt{2}}} \frac{1}{u}\,du$$

$$= -2\left[\ln|u|\right]_2^{\frac{\sqrt{2}+1}{\sqrt{2}}}$$

$$= -2\left[\ln\frac{\sqrt{2}+1}{\sqrt{2}} - \ln 2\right]$$

$$= -2\ln\frac{\sqrt{2}+1}{2\sqrt{2}}$$

$$= \ln\left(\frac{2\sqrt{2}}{\sqrt{2}+1}\right)^2$$

$$= \ln\frac{8}{3+2\sqrt{2}}$$

Question 7. *(3 marks)* Use the Second Fundamental Theorem of Calculus to find $F'(x)$.

$$f(g(x)) = F(x) = \int_0^{\sin 5x} \frac{1}{\sqrt{1-y^2}}\,dy$$

Let $g(x) = \sin 5x \Rightarrow g'(x) = 5\cos 5x$

$$f(x) = \int_0^{x} \frac{1}{\sqrt{1-y^2}}\,dy \Rightarrow f'(x) = \frac{1}{\sqrt{1-x^2}}$$

(by the second fundamental thm of calculus)

$$\frac{d}{dx}\left[f(g(x))\right] = f'(g(x))g'(x)$$

$$= \frac{1}{\sqrt{1-\sin^2 5x}}\, 5\cos 5x$$

$$= \frac{1}{\sqrt{\cos^2 5x}}\, 5\cos 5x$$

$$= \frac{5\cancel{\cos 5x}}{\cancel{\cos 5x}} = 5$$

Question 8. *(5 marks)* Integrate the following indefinite integral:

$$\int \frac{(\ln x)^3}{x}\,dx \overset{①,②}{=} \int \frac{u^3}{x}\,x\,du$$

① $u = \ln x$

$du = \frac{dx}{x}$

② $x\,du = dx$

$$= \int u^3\,du$$

$$= \frac{u^4}{4} + C$$

$$= \frac{(\ln x)^4}{4} + C$$

Question 9. *(5 marks)* Integrate the following indefinite integral:

$$\int \frac{e^{\sqrt[3]{x}}}{\sqrt[3]{x^2}}\,dx = \int \frac{e^{x^{1/3}}}{x^{2/3}}\,dx$$

① $u = x^{1/3}$

$du = \frac{dx}{3x^{2/3}}$

② $3x^{2/3}du = dx$

$$\overset{①,②}{=} \int \frac{e^u}{\cancel{x^{2/3}}} \cancel{3x^{2/3}}\,du$$

$$= 3\int e^u\,du$$

$$= 3e^u + C$$

$$= 3e^{x^{1/3}} + C$$

Question 10. *(5 marks)* Evaluate the following definite integral:

$$\int_{-3}^{-2} \frac{x^2+x}{2x^3+3x^2+1}\,dx$$

① $u = 2x^3 + 3x^2 + 1$

$du = 6x^2 + 6x\,dx$

② $\frac{du}{6x^2+6x} = dx$

$u(-3) = 2(-3)^3 + 3(-3)^2 + 1$

$= -26$

$u(-2) = 2(-2)^3 + 3(-2)^2 + 1$

$= -3$

$$\overset{①,②}{=} \int_{-26}^{-3} \frac{x^2+x}{u} \frac{du}{6x^2+6x}$$

$$= \frac{1}{6}\int_{-26}^{-3} \frac{\cancel{x^2+x}}{u} \frac{du}{\cancel{x^2+x}}$$

$$= \frac{1}{6}\Big[\ln|u|\Big]_{-26}^{-3}$$

$$= \frac{1}{6}\left[\ln|-3| - \ln|-26|\right]$$

$$= \frac{1}{6}\left[\ln 3 - \ln 26\right]$$

$$= \ln \sqrt[6]{3/26}$$

Bonus Question. *(3 marks)*
Integrate the following indefinite intergral:

$$\int \frac{1}{x(\operatorname{arcsec} x)(\ln \operatorname{arcsec} x)\sqrt{x^2-1}}\,dx \overset{①,②}{=} \int \frac{1}{u}\,du$$

① $u = \ln \operatorname{arcsec} x$

② $du = \frac{1}{(\operatorname{arcsec} x)x\sqrt{x^2-1}}\,dx$

$$= \ln|u| + C$$

$$= \ln|\ln \operatorname{arcsec} x| + C$$