

Name: \_\_\_\_\_  
Student ID: \_\_\_\_\_

## Test 2

This test is graded out of 45 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

**Question 1.** (5 marks) Find the average of the function  $f(x) = \frac{x(x^2-1)}{x^2+1}$  over the interval  $[-1, 1]$ .

**Question 2.** (5 marks) Evaluate the following definite integral:

$$\int_0^{\frac{\pi}{2}} \frac{\cos x}{1 + \sin^2 x} dx$$

**Question 3.** (2 marks for sketch and 3 marks for area) Sketch the graph of the two following algebraic functions  $f(x) = \sqrt{3x+1}$  and  $g(x) = x+1$  and find the area bounded by the two functions.

**Question 4.** (5 marks) Find the volume of the solid of revolution generated by the bounded region of the functions  $f(x) = -x^2 + 6x$  and  $g(x) = x^2 - 2x$  revolved about the y-axis.

**Question 5.** (5 marks) Find the volume of the solid of revolution generated by the bounded region of the functions  $f(x) = -x + 4$ ,  $g(x) = x$  and  $y = 0$  revolved about y-axis.

**Question 6.** (5 marks) Find the arc length of the graph of the function  $f(x) = \frac{3}{2}x^{2/3}$  over the interval  $[1, 8]$ .

**Question 7.** (5 marks) Use the Trapezoidal Rule with  $n = 4$  to approximate the value of the definite integral and compare your answer to the exact value of the definite integral. (i.e. calculate the definite integral using the Fundamental Theorem of Calculus.)

$$\int_0^2 xe^{x^2} dx$$

**Question 8.** (5 marks) A conical tank with its tip pointing upwards is filled by a pump which is located 5m below the tank. If the fluid that has a density of  $\rho = 2000 \frac{\text{kg}}{\text{m}^3}$  and the tank is 4m across at the bottom and 3m in height, how much work is required to fill the tank?

**Question 9.** (5 marks) Find the 'c' value(s) guaranteed by the Mean Value Theorem for Integrals for the function  $f(x) = \tan x$  over  $[\frac{\pi}{6}, \frac{\pi}{3}]$ .

**Bonus Question.** (5 marks)

Prove one of the following statement:

- If  $f(x)$  is an even function then

$$\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$$

- The formula of the arc length of a function.