Name: _____

Student ID: _____

Test 2

This test is graded out of 45 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* Find the average of the function $f(x) = \frac{x(x^2-2)}{x^2+2}$ over the interval $[-2,2]$.

Question 2. *(5 marks)* Evaluate the following definite integral:

$$\int_{\frac{\pi}{2}}^{\pi} \frac{\sin x}{1+\cos^2 x}\, dx$$

Question 3. *(2 marks for sketch and 3 marks for area)* Sketch the graph of the two following algebraic functions $f(x) = \sqrt{4x} + 2$ and $g(x) = x + 2$ and find the area bounded by the two functions.

Question 4. *(5 marks)* Find the volume of the solid of revolution generated by the bounded region of the functions $f(x) = x^2$ and $g(x) = -x^2 + 8$ revolved about the x-axis.

Question 5. *(5 marks)* Find the volume of the solid of revolution generated by the bounded region of the functions $f(x) = -x + 8$, $g(x) = x$ and $y = 0$ revolved about x-axis.

Question 6. *(5 marks)* Find the arc length of the graph of the function $f(x) = \frac{x^3}{6} + \frac{1}{2x}$ over the interval $[1,2]$.

Question 7. *(5 marks)* Use the Simpson's Rule with $n = 4$ to approximate the value of the definite integral and compare your answer to the exact value of the definite integral. *(i.e. calculate the definite integral using the Fundamental Theorem of Calculus.)*

$$\int_0^2 xe^{x^2} \, dx$$

Question 8. *(5 marks)* A conical tank with its tip pointing upwards is emptied by a pump which is located 2m above the tank. If the fluid that has a density of $\rho = 2000\frac{\text{kg}}{\text{m}^3}$ and the tank is 6m across at the bottom and 4m in height, how much work is required to empty the tank?

Question 9. *(5 marks)* Find the 'c' value(s) guaranteed by the Mean Value Theorem for Integrals for the function $f(x) = \cot x$ over $\left[\frac{\pi}{6}, \frac{\pi}{4}\right]$.

Bonus Question. *(5 marks)*
Prove one of the following statement:

- If $f(x)$ is an even function then

$$\int_{-a}^{a} f(x)\, dx = 2 \int_{0}^{a} f(x)\, dx$$

- The formula of the arc length of a function.