Dawson College: **Calculus II**: 201-NYB-05-S2: Summer 2008

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Test 3

This test is graded out of 45 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* Integrate the following indefinite integral:

$$\int_{\pi/3}^{\pi/4} x\sec^2 x\,dx = \Big[uv\Big]_{\pi/3}^{\pi/4} - \int_{\pi/3}^{\pi/4} v\,du$$

$u = x$ $\quad du = dx$

$v = \tan x$ $\quad dv = \sec^2 x\,dx$

$$= \Big[x\tan x\Big]_{\pi/3}^{\pi/4} - \int_{\pi/3}^{\pi/4} \tan x\,dx$$

$$= \frac{\pi}{4}\tan\frac{\pi}{4} - \frac{\pi}{3}\tan\frac{\pi}{3} + \Big[\ln|\cos x|\Big]_{\pi/3}^{\pi/4}$$

$$= \frac{\pi}{4} - \frac{\pi}{3}\sqrt{3} + \ln\left|\cos\frac{\pi}{4}\right| - \ln\left|\cos\frac{\pi}{3}\right|$$

$$= \frac{\pi}{4} - \frac{\pi}{3}\sqrt{3} + \ln\frac{1}{\sqrt{2}} - \ln\frac{1}{2}$$

$$= \frac{\pi}{4} - \frac{\pi}{3}\sqrt{3} + \ln\frac{2}{\sqrt{2}}$$

Question 2. *(5 marks)* Integrate the following indefinite integral:

$$\int \cos^4 x\,dx = \int \left(\frac{1+\cos 2x}{2}\right)^2 dx$$

$$= \frac{1}{4}\int (1+\cos 2x)^2\,dx$$

$$= \frac{1}{4}\int 1 + 2\cos 2x + \cos^2 2x\,dx$$

$$= \frac{1}{4}\int 1 + 2\cos 2x + \frac{1}{2} + \frac{\cos 4x}{2}\,dx$$

$$= \frac{1}{4}\left[\frac{3}{2}x + \sin 2x + \frac{\sin 4x}{8}\right] + C$$

Question 3. *(5 marks)* Integrate the following indefinite integral:

$$\int \frac{\sqrt{x^2-4}}{x}\,dx$$

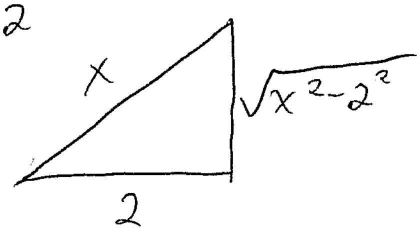
$x = 2\sec\theta$

$dx = 2\sec\theta\tan\theta\,d\theta$

$$= \int \frac{\sqrt{(2\sec\theta)^2 - 4}}{\cancel{2\sec\theta}}\,\cancel{2\sec\theta}\tan\theta\,d\theta$$

$$= \int \sqrt{4(\sec^2\theta - 1)}\,\tan\theta\,d\theta$$

$\frac{h}{a} = \frac{x}{2} = \sec\theta$



$$= \int \sqrt{4\tan^2\theta}\,\tan\theta$$

$$= \int 2\tan^2\theta\,d\theta$$

$\tan\theta = \frac{o}{a} = \frac{\sqrt{x^2-2^2}}{2}$

$$= 2\int \sec^2\theta - 1\,d\theta$$

$\operatorname{arcsec}\frac{x}{2} = \theta$

$$= 2\tan\theta - 2\theta + C$$

$$= \sqrt{x^2-2^2} - 2\operatorname{arcsec}\frac{x}{2} + C$$

Question 4. *(5 marks)* Integrate the following indefinite integral:

$$\int \frac{x+1}{(x-1)(x^2+1)}\,dx$$

Lets use partial fractions.

$$\frac{x+1}{(x-1)(x^2+1)} = \frac{A}{(x-1)} + \frac{Bx+C}{x^2+1}$$

$$x+1 = A(x^2+1) + (Bx+C)(x-1)$$

Let $x=1$

$$1+1 = A(1^2+1) + (B(1)+C)(1-1)$$

$$1 = A$$

Let $x=0$

$$0+1 = A(0^2+1) + (B(0)+C)(0-1)$$

$$0 = C$$

Let $x=-1$

$$-1+1 = 1((-1)^2+1) + (B(-1)+(0))(-1-1)$$

$$0 = 2 + 2B$$

$$-1 = B$$

$$\therefore \int \frac{x+1}{(x-1)(x^2+1)}\,dx = \int \frac{1}{x-1} + \frac{-x}{x^2+1}\,dx$$

$$= \ln|x-1| - \int \frac{x}{x^2+1}\,dx$$

$$= \ln|x-1| - \frac{1}{2}\ln|x^2+1| + C$$

$$= \ln|x-1| - \frac{1}{2}\ln|x^2+1| + C$$

Question 5. *(5 marks)* Evaluate the limit, using L'Hôpital's Rule if necessary.

$$y = \lim_{x\to\infty} (1+x)^{2/x}$$

$$\ln y = \ln \lim_{x\to\infty} (1+x)^{2/x}$$

$$\ln y = \lim_{x\to\infty} \ln (1+x)^{2/x}$$

$$\ln y = \lim_{x\to\infty} \frac{2}{x} \ln(1+x) \quad \text{has IF } \frac{\infty}{\infty}$$

$$\ln y = \lim_{x\to\infty} \frac{\frac{2}{1+x}}{1}$$

$$\ln y = \lim_{x\to\infty} \frac{2}{1+x}$$

$$\ln y = 0$$

$$e^{\ln y} = e^{0}$$

$$y = 1$$

Question 6. *(5 marks)* Solve the following improper integral:

$$\int_0^\infty xe^x \, dx$$

$u = x \qquad du = dx$

$v = e^x \qquad dv = e^x dx$

$$= \lim_{b\to\infty} \int_0^b xe^x dx$$

$$= \lim_{b\to\infty} \left[\left[uv\right]_0^b - \int_0^b v\,du \right]$$

$$= \lim_{b\to\infty} \left[\left[xe^x\right]_0^b - \int_0^b e^x dx \right]$$

$$= \lim_{b\to\infty} \left[\overset{\infty}{\cancel{be^b}} - 0 - \overset{\infty}{\cancel{e^b}} + \overset{1}{\cancel{e^0}} \right]$$

$\therefore$ does not converge.

Question 7. *(5 marks)* Solve the following improper integral:

$$\int_0^1 \frac{9}{\sqrt{1-x}}\,dx$$

$$= \lim_{b \to 1} \int_0^b \frac{9}{\sqrt{1-x}}\,dx$$

$$= \lim_{b \to 1} \Big[9(-2)\sqrt{1-x} \Big]_0^b$$

$$= \lim_{b \to 1} \cancel{-18\sqrt{1-b}}^{\,0} + 18\sqrt{1-0}$$

$$= 18$$

Question 8. *(5 marks)* Integrate the following indefinite integral:

$$\int \frac{1}{e^{2x}-1}\,dx$$

$$= \int \frac{1}{e^{2x}-1}\left(\frac{e^{-2x}}{e^{-2x}}\right) dx$$

$$= \int \frac{e^{-2x}}{1-e^{-2x}}\,dx \qquad u = 1-e^{-2x}$$

$$du = 2e^{-2x}\,dx$$

$$= \frac{1}{2}\int \frac{du}{u}$$

$$= \frac{1}{2}\ln|u| + C$$

$$= \frac{1}{2}\ln|1-e^{-2x}| + C$$

Question 9. *(5 marks)* Determine the convergence or divergence of the sequence with the given n^{th} term. If the sequence converges find its limit.

$$b_n = \frac{n^2}{e^n}$$

$\lim_{n \to \infty} \frac{n^2}{e^n}$ has IF $\frac{\infty}{\infty}$ so we use l'Hôpital's Rule

$= \lim_{n \to \infty} \frac{2n}{e^n}$ has IF $\frac{\infty}{\infty}$ so we use l'Hopital's Rule again.

$= \lim_{n \to \infty} \frac{2}{e^n}$

$= 0$

Bonus Question. *(3 marks)*

$$\int \frac{\sqrt{1-x}}{\sqrt{x}}\, dx$$

$$\int \frac{\sqrt{1-x}}{\sqrt{x}}\,dx$$

let $u = \sqrt{x}$

$du = \frac{dx}{2\sqrt{x}}$

$2\sqrt{x}\,du = dx$

$$= \int \frac{\sqrt{1-(\sqrt{x})^2}}{\sqrt{x}}\,dx$$

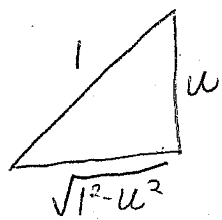
$$= \int \frac{\sqrt{1-u^2}}{\cancel{\sqrt{x}}}\,2\cancel{\sqrt{x}}\,du$$

$$= 2\int \sqrt{1-u^2}\,du$$

$u = \sin\theta$

$du = \cos\theta\,d\theta$

$$= 2\int \sqrt{1-\sin^2\theta}\,\cos\theta\,d\theta$$



$$= 2\int \sqrt{\cos^2\theta}\,\cos\theta\,d\theta$$

$\therefore \cos\theta = \sqrt{1-u^2}$

$$= 2\int \cos^2\theta\,d\theta$$

$$= 2\int \frac{1+\cos 2\theta}{2}\,d\theta$$

$$= \int 1+\cos 2\theta\,d\theta$$

$$= \theta + \frac{\sin 2\theta}{2} + C$$

$$= \theta + \frac{2\sin\theta\cos\theta}{2} + C$$

$$= \arcsin u + \sin\theta\cos\theta + C$$

$$= \arcsin\sqrt{x} + u\sqrt{1-u^2} + C$$

$$= \arcsin\sqrt{x} + \sqrt{x}\sqrt{1-x} + C$$