LAST NAME: SOLUTIONS

FIRST NAME: _____

STUDENT NUMBER: _____

QUIZ 2 (A)

DAWSON COLLEGE

201-NYC-05 - Linear Algebra

Instructor: E. Richer

Date: June 26th 2008

Question 1. *(5 marks)*

Find $\det(A)$ where

$$A = \begin{bmatrix} 2 & 1 & 0 & 0 \\ -2 & 1 & -2 & 0 \\ 3 & 0 & 0 & 2 \\ 2 & -1 & 3 & 0 \end{bmatrix}$$

$$\det A = -2 \det \begin{bmatrix} 2 & 1 & 0 \\ -2 & 1 & -2 \\ 2 & -1 & 3 \end{bmatrix} \quad \text{(EXPANSION ALONG COLUMN 4)}$$

$$= -2 \left(2 \det \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix} - \det \begin{bmatrix} -2 & -2 \\ 2 & 3 \end{bmatrix} \right) \quad \text{(EXP. ALONG ROW 1)}$$

$$= -2 \left(2(1) - (-2) \right)$$

$$= -2(4) = -8$$

Question 2. *(5 marks)*
Determine which of the following matrices are invertible. If the matrix is invertible find its inverse.

$$A = \begin{bmatrix} \frac{1}{2} & 0 & 1 \\ 0 & -1 & 3 \\ 0 & 0 & 3 \end{bmatrix} \quad B = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 3 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 3 & 4 \end{bmatrix}$$

A & B are invertible
C is not. We know this from looking at the main diagonal, since the matrices are triangular (or diagonal).

A^{-1}:

$$\left[\begin{array}{ccc|ccc} \frac{1}{2} & 0 & 1 & 1 & 0 & 0 \\ 0 & -1 & 3 & 0 & 1 & 0 \\ 0 & 0 & 3 & 0 & 0 & 1 \end{array}\right] \quad R_2 - R_3 \to R_2 \quad \left[\begin{array}{ccc|ccc} \frac{1}{2} & 0 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 3 & 0 & 0 & 1 \end{array}\right]$$

$$R_1 - \tfrac{1}{3}R_3 \to R_1 \quad \left[\begin{array}{ccc|ccc} \frac{1}{2} & 0 & 0 & 1 & 0 & -\frac{1}{3} \\ 0 & -1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 3 & 0 & 0 & 1 \end{array}\right]$$

$2R_1 \to R_1$
$-1R_2 \to R_2$
$\frac{1}{3}R_3 \to R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 0 & -2/3 \\ 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1/3 \end{array}\right] \quad A^{-1} = \begin{bmatrix} 2 & 0 & -2/3 \\ 0 & -1 & 1 \\ 0 & 0 & 1/3 \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} -1/2 & 0 & 0 \\ 0 & 1/5 & 0 \\ 0 & 0 & 1/3 \end{bmatrix}$$

Question 3. *(5 marks)*

Find A^{-1} using $\text{adj}(A)$, where $A = \begin{bmatrix} 1 & 0 & 1 \\ -2 & 1 & -2 \\ 3 & 0 & 0 \end{bmatrix}$

$$C_{11} = \det\begin{bmatrix} 1 & -2 \\ 0 & 0 \end{bmatrix} = 0$$

$$C_{12} = -\det\begin{bmatrix} -2 & -2 \\ 3 & 0 \end{bmatrix} = -6$$

$$C_{13} = \det\begin{bmatrix} -2 & 1 \\ 3 & 0 \end{bmatrix} = -3$$

$$C_{21} = -\det\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = 0$$

$$C_{22} = \det\begin{bmatrix} 1 & 1 \\ 3 & 0 \end{bmatrix} = -3$$

$$C_{23} = -\det\begin{bmatrix} 1 & 0 \\ 3 & 0 \end{bmatrix} = 0$$

$$C_{31} = \det\begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix} = -1$$

$$C_{32} = -\det\begin{bmatrix} 1 & 1 \\ -2 & -2 \end{bmatrix} = 0$$

$$C_{33} = \det\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} = 1$$

$$\det A = 3\det\begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix} = -3$$

MATRIX OF COFACTORS $\begin{bmatrix} 0 & -6 & -3 \\ 0 & -3 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

$$A^{-1} = \frac{1}{\det A}\,\text{adj}\,A = \frac{1}{-3}\begin{bmatrix} 0 & 0 & -1 \\ -6 & -3 & 0 \\ -3 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1/3 \\ 2 & 1 & 0 \\ 1 & 0 & -1/3 \end{bmatrix}$$

Question 4. *(5 marks)*
Use Cramer's Rule to find the value of x_2 in the following system of linear equations.

$-2x_1 + 3x_3 = 1$
$x_1 + 2x_2 = 2$
$4x_1 + x_2 + x_3 = 0$

$Ax = b$ where

$$A = \begin{bmatrix} -2 & 0 & 3 \\ 1 & 2 & 0 \\ 4 & 1 & 1 \end{bmatrix} \qquad b = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -2 & 1 & 3 \\ 1 & 2 & 0 \\ 4 & 0 & 1 \end{bmatrix}$$

$$\det A = -2\det\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} + 3\det\begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix}$$

$$= -4 + 3(-7) = -25$$

$$\det A_2 = 4\det\begin{bmatrix} 1 & 3 \\ 2 & 0 \end{bmatrix} + \det\begin{bmatrix} -2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$= 4(-6) - 5 = -29$$

$$x_2 = \frac{\det A_2}{\det A} = 29/25$$