LAST NAME: SOLUTIONS

FIRST NAME: _____

STUDENT NUMBER: _____

QUIZ 2 (B)

DAWSON COLLEGE

201-NYC-05 - Linear Algebra

Instructor: E. Richer

Date: June 26th 2008

Question 1. *(5 marks)*

Find $\det(A)$ where

$$A = \begin{bmatrix} 0 & 1 & 0 & 2 \\ 0 & 1 & -2 & -2 \\ 2 & 0 & 0 & 3 \\ 0 & -1 & 3 & 2 \end{bmatrix}$$

$$\det A = 2 \det \begin{bmatrix} 1 & 0 & 2 \\ 1 & -2 & -2 \\ -1 & 3 & 2 \end{bmatrix}$$ (Expansion Along column 1)

$$= 2\left(\det \begin{bmatrix} -2 & -2 \\ 3 & 2 \end{bmatrix} + 2 \det \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}\right)$$ (Expansion Along Row 1)

$$= 2\left(2 + 2(1)\right) = 2(4) = 8$$

Question 2. *(5 marks)*
Determine which of the following matrices are invertible. If the matrix is invertible find its inverse.

$$A = \begin{bmatrix} \frac{1}{2} & 0 & 1 \\ 0 & -1 & 2 \\ 0 & 0 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 3 & 4 \end{bmatrix} \quad C = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

THESE ARE diAgonAl & triAngulAr MAtrices, so by inspection of the MAin diAgonAl we observe that A, C ARE invertible
B is not

A^{-1} :

$$\left[\begin{array}{ccc|ccc} 1/2 & 0 & 1 & 1 & 0 & 0 \\ 0 & -1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 \end{array}\right] \quad R_2 - R_3 \to R_2 \quad \left[\begin{array}{ccc|ccc} 1/2 & 0 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$R_1 - \tfrac{1}{2} R_3 \to R_1 \quad \left[\begin{array}{ccc|ccc} 1/2 & 0 & 0 & 1 & 0 & -1/2 \\ 0 & -1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 2 & 0 & 0 & 1 \end{array}\right]$$

$2R_1 \to R_1$
$-1R_2 \to R_2$
$\tfrac{1}{2}R_3 \to R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1/2 \end{array}\right] \quad A^{-1} = \begin{bmatrix} 2 & 0 & -1 \\ 0 & -1 & 1 \\ 0 & 0 & 1/2 \end{bmatrix}$$

$$C^{-1} = \begin{bmatrix} -1/2 & 0 & 0 \\ 0 & 1/4 & 0 \\ 0 & 0 & -1/3 \end{bmatrix}$$

B^{-1} does not exist

Question 3. *(5 marks)*

Find A^{-1} using $\text{adj}(A)$, where $A = \begin{bmatrix} 1 & 0 & 1 \\ -2 & 1 & -2 \\ 3 & 0 & 0 \end{bmatrix}$

See version A

Question 4. *(5 marks)*
Use Cramer's Rule to find the value of $\mathbf{x_1}$ in the following system of linear equations.

$-2x_1 + 3x_3 = 1$
$x_1 + 2x_2 = 2$
$4x_1 + x_2 + x_3 = 0$

$$A = \begin{bmatrix} -2 & 0 & 3 \\ 1 & 2 & 0 \\ 4 & 1 & 1 \end{bmatrix}$$

See version A

$\det A = -25$

$$A_1 = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 2 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\det A_1 = \det \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} + 3 \det \begin{bmatrix} 2 & 2 \\ 0 & 1 \end{bmatrix}$$

$$= 2 + 3(2) = 8$$

$$x_1 = \frac{\det A_1}{\det A} = 8/-25$$