LAST NAME: SOLUTIONS

FIRST NAME: ______________

STUDENT NUMBER: ______________

QUIZ 2 (A)

DAWSON COLLEGE

201-NYC-05-S2 - Linear Algebra

Instructor: E. Richer

Date: June 26th 2008

Question 1. *(5 marks)*

Find $\det(A)$ where

$$A = \begin{bmatrix} 0 & 1 & 0 & 2 \\ 0 & 1 & -4 & -2 \\ -2 & 0 & 0 & 3 \\ 0 & -1 & 6 & 2 \end{bmatrix}$$

$$\det A = -2 \det \begin{bmatrix} 1 & 0 & 2 \\ 1 & -4 & -2 \\ -1 & 6 & 2 \end{bmatrix}$$

$$= -2 \left(\det \begin{bmatrix} -4 & -2 \\ 6 & 2 \end{bmatrix} + 2 \det \begin{bmatrix} 1 & -4 \\ -1 & 6 \end{bmatrix} \right)$$

$$= -2 \left(4 + 2(2) \right)$$

$$= -2(8) = -16$$

Question 2. *(5 marks)*
Determine which of the following matrices are invertible. If the matrix is invertible find its inverse.

$$A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -5 \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{1}{2} & 0 & 1 \\ 0 & -1 & 3 \\ 0 & 0 & 3 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 3 & 4 \end{bmatrix}$$

These are diagonal or triangular, so by inspection of the main diagonal

B, A is invertible

C is not

$$A^{-1} = \begin{bmatrix} -1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & -1/5 \end{bmatrix}$$

B^{-1}

$$\left[\begin{array}{ccc|ccc} 1/2 & 0 & 1 & 1 & 0 & 0 \\ 0 & -1 & 3 & 0 & 1 & 0 \\ 0 & 0 & 3 & 0 & 0 & 1 \end{array}\right]$$

$R_1 - 1/3 R_3 \to R_1$
$R_2 - R_3 \to R_2$

$$\left[\begin{array}{ccc|ccc} 1/2 & 0 & 0 & 1 & 0 & -1/3 \\ 0 & -1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 3 & 0 & 0 & 1 \end{array}\right]$$

$2R_1 \to R_1$
$-1 R_2 \to R_2$
$1/3 R_3 \to R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 0 & -2/3 \\ 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1/3 \end{array}\right]$$

$$B^{-1} = \begin{bmatrix} 2 & 0 & -2/3 \\ 0 & -1 & 1 \\ 0 & 0 & 1/3 \end{bmatrix}$$

Question 3. *(5 marks)*

Suppose $x^2 - 3 = 1$, find A^{-1}, where $A = \begin{bmatrix} x & 1 & 0 \\ 3 & x & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(Hint: Use the matrix of cofactors and the adjoint of A)

$$\det A = \det \begin{bmatrix} x & 1 \\ 3 & x \end{bmatrix} = x^2 - 3 = 1$$

$$C_{11} = \det \begin{bmatrix} x & 0 \\ 0 & 1 \end{bmatrix} = x$$

$$C_{12} = -\det \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} = -3$$

$$C_{13} = \det \begin{bmatrix} 3 & x \\ 0 & 0 \end{bmatrix} = 0$$

$$C_{21} = -\det \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = -1$$

$$C_{22} = \det \begin{bmatrix} x & 0 \\ 0 & 1 \end{bmatrix} = x$$

$$C_{23} = -\det \begin{bmatrix} x & 1 \\ 0 & 0 \end{bmatrix} = 0$$

$$C_{31} = \det \begin{bmatrix} 1 & 0 \\ x & 0 \end{bmatrix} = 0$$

$$C_{32} = -\det \begin{bmatrix} x & 0 \\ 3 & 0 \end{bmatrix} = 0$$

$$C_{33} = \det \begin{bmatrix} x & 1 \\ 3 & x \end{bmatrix} = x^2 - 3 = 1$$

MATRIX OF COFACTORS IS

$$\begin{bmatrix} x & -3 & 0 \\ -1 & x & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det A} \operatorname{adj} A$$

$$= \frac{1}{1} \begin{bmatrix} x & -1 & 0 \\ -3 & x & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Question 4. *(5 marks)*
Use Cramer's Rule to find the value of $\mathbf{x_1}$ in the following system of linear equations.

$-2x_1 + 3x_3 = 0$
$x_1 + 2x_2 = 1$
$4x_1 + x_2 + x_3 = 1$

$Ax = b$ where

$$A = \begin{bmatrix} -2 & 0 & 3 \\ 1 & 2 & 0 \\ 4 & 1 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$\det A = -2\det\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} + 3\det\begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix}$$

$$= -2(2) + 3(1-8) = -4-21 = -25$$

$$A_1 = \begin{bmatrix} 0 & 0 & 3 \\ 1 & 2 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\det A_1 = 3\det\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} = 3(1-2) = -3$$

so $x_1 = \dfrac{\det A_1}{\det A} = \dfrac{-3}{-25} = 3/25$