LAST NAME: SOLUTIONS

FIRST NAME: ______________

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QUIZ 2 (B)

DAWSON COLLEGE

201-NYC-05-S2 - Linear Algebra

Instructor: E. Richer

Date: June 26th 2008

Question 1. *(5 marks)*

Find $\det(A)$ where

$$A = \begin{bmatrix} 0 & 2 & 0 & 2 \\ 0 & 2 & -2 & -2 \\ 2 & 0 & 0 & 3 \\ 0 & -2 & 3 & 2 \end{bmatrix}$$

$$\det A = 2 \det \begin{bmatrix} 2 & 0 & 2 \\ 2 & -2 & -2 \\ -2 & 3 & 2 \end{bmatrix}$$

$$= 2\left(2 \det \begin{bmatrix} -2 & -2 \\ 3 & 2 \end{bmatrix} + 2 \det \begin{bmatrix} 2 & -2 \\ -2 & 3 \end{bmatrix} \right)$$

$$= 2\left(2(2) + 2(2) \right)$$

$$= 2(8) = 16$$

Question 2. *(5 marks)*
Determine which of the following matrices are invertible. If the matrix is invertible find its inverse.

$$A = \begin{bmatrix} \frac{1}{2} & 0 & 1 \\ 0 & -1 & 2 \\ 0 & 0 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 3 & 4 \end{bmatrix} \quad C = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

These are triangular matrices, so by inspection of the main diagonals A, C are invertible B is not (it has a zero on the diagonal)

A^{-1} :

$$\left[\begin{array}{ccc|ccc} \frac{1}{2} & 0 & 1 & 1 & 0 & 0 \\ 0 & -1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 \end{array}\right] \quad \begin{array}{l} R_1 - \frac{1}{2}R_3 \to R_1 \\ R_2 - R_3 \to R_2 \end{array}$$

$$\left[\begin{array}{ccc|ccc} \frac{1}{2} & 0 & 0 & 1 & 0 & -\frac{1}{2} \\ 0 & -1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 2 & 0 & 0 & 1 \end{array}\right] \quad \begin{array}{l} 2R_1 \to R_1 \\ -1R_2 \to R_2 \\ \frac{1}{2}R_3 \to R_3 \end{array} \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 0 & 0 & \frac{1}{2} \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 2 & 0 & -1 \\ 0 & -1 & 1 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$$

$$C^{-1} \text{ (diagonal)} = \begin{bmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{4} & 0 \\ 0 & 0 & -\frac{1}{3} \end{bmatrix}$$

Question 3. *(5 marks)*

Suppose $x^2 - 2 = 1$, find A^{-1}, where $A = \begin{bmatrix} x & 1 & 0 \\ 2 & x & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(Hint: Use the matrix of cofactors and the adjoint of A)

$$\det A = \det \begin{bmatrix} x & 1 \\ 2 & x \end{bmatrix} = x^2 - 2 = 1$$

$$C_{11} = \det \begin{bmatrix} x & 0 \\ 0 & 1 \end{bmatrix} = x \qquad C_{21} = -\det \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = -1$$

$$C_{12} = -\det \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} = -2 \qquad C_{22} = \det \begin{bmatrix} x & 0 \\ 0 & 1 \end{bmatrix} = x$$

$$C_{13} = \det \begin{bmatrix} 2 & x \\ 0 & 0 \end{bmatrix} = 0 \qquad C_{23} = -\det \begin{bmatrix} x & 1 \\ 0 & 0 \end{bmatrix} = 0$$

$$C_{31} = \det \begin{bmatrix} 1 & 0 \\ x & 0 \end{bmatrix} = 0$$

$$C_{32} = -\det \begin{bmatrix} x & 0 \\ 2 & 0 \end{bmatrix} = 0$$

$$C_{33} = \det \begin{bmatrix} x & 1 \\ 2 & x \end{bmatrix} = x^2 - 2 = 1$$

MATRIX OF COFACTORS is $\begin{bmatrix} x & -2 & 0 \\ -1 & x & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$A^{-1} = \frac{1}{\det A} \operatorname{Adj} A = \frac{1}{1} \begin{bmatrix} x & -1 & 0 \\ -2 & x & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} x & -1 & 0 \\ -2 & x & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Question 4. *(5 marks)*
Use Cramer's Rule to find the value of $\mathbf{x_1}$ in the following system of linear equations.

$-2x_1 + 3x_3 = 1$
$x_1 + 2x_2 = 2$
$4x_1 + x_2 + x_3 = 0$

$$Ax = b$$

$$A = \begin{bmatrix} -2 & 0 & 3 \\ 1 & 2 & 0 \\ 4 & 1 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

$$\det A = -2\det\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} + 3\det\begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix}$$
$$= -2(2) + 3(-7) = -4 - 21 = -25$$

$$A_1 = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 2 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\det A_1 = \det\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} + 3\det\begin{bmatrix} 2 & 2 \\ 0 & 1 \end{bmatrix}$$
$$= 2 + 3(2) = 8$$

$$x_1 = \frac{\det A_1}{\det A} = 8/-25$$