LAST NAME: SOLUTIONS

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QUIZ 3 (A)

DAWSON COLLEGE

201-NYC-05-S2 Linear Algebra

Instructor: E. Richer

Date: July 10th 2008

Question 1. *(10 marks)*

Let $\vec{u} = (1,2,-1)$, $\vec{v} = (1,1,-1)$ and $\vec{w} = (0,1,1)$.

Compute the following:

(i) $||2\vec{v}||$

(ii) $\text{proj}_{\vec{v}}\vec{u}$

(iii) $(\vec{u}\times\vec{v})\times\vec{w}$

(iv) $3\vec{u}\cdot\vec{w}$

(v) The volume of the parrallelepiped defined by the vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$

(i) $||2\vec{v}|| = 2||\vec{v}|| = 2\sqrt{1^2+(1)^2+(-1)^2} = \boxed{2\sqrt{3}}$

(ii) $\text{proj}_{\vec{v}}\vec{u} = \left(\dfrac{\vec{v}\cdot\vec{u}}{\vec{v}\cdot\vec{v}}\right)\vec{v} = \left(\dfrac{4}{3}\right)(1,1,-1) = \boxed{(4/3,\ 4/3,\ -4/3)}$

(iii) $\vec{u}\times\vec{v} = (-1,0,-1)$ $\quad \begin{pmatrix}1\\2\\-1\end{pmatrix}\begin{pmatrix}1\\1\\-1\end{pmatrix}$

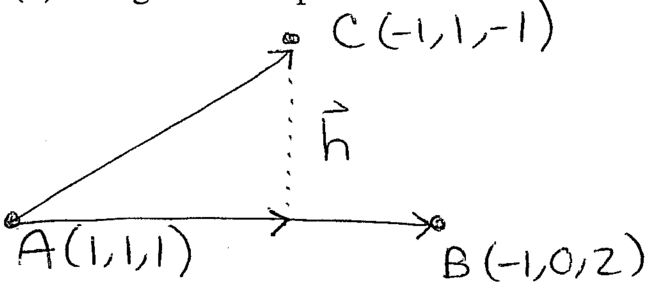
$(\vec{u}\times\vec{v})\times\vec{w} = \boxed{(1,\ 1,\ -1)}$ $\quad \begin{pmatrix}-1\\0\\-1\end{pmatrix}\begin{pmatrix}0\\1\\1\end{pmatrix}$

(iv) $3\vec{u}\cdot\vec{W} = 3(\vec{u}\cdot\vec{W})$
$= 3(1) = \boxed{3}$

(v) $V = |\vec{u}\cdot(\vec{v}\times\vec{w})|$
$= |(1,2,-1)\cdot(2,-1,1)|$
$= |-1| = \boxed{1}$

$\vec{v}\times\vec{w} = (2,-1,1)$ $\quad \begin{pmatrix}1\\1\\-1\end{pmatrix}\begin{pmatrix}0\\1\\1\end{pmatrix}$

Question 2. *(10 marks)*
A triangle is defined by the points $A(1,1,1)$, $B(-1,0,2)$ and $C(-1,1,-1)$. Compute the **area of the triangle** in two different ways:
(i) Using projections
(ii) Using the cross product



$$\vec{AC} = (-2,0,-2)$$
$$\vec{AB} = (-2,-1,1)$$

$$\text{proj}_{\vec{AB}} \vec{AC} = \frac{\vec{AB} \cdot \vec{AC}}{\vec{AB} \cdot \vec{AB}} \left(\vec{AB}\right)$$
$$= \frac{2}{6}(-2,-1,1) = \left(-2/3, -1/3, 1/3\right)$$

$$\vec{h} = \vec{AC} - \text{proj}_{\vec{AB}} \vec{AC}$$
$$= (-2,0,-2) - (-2/3, -1/3, 1/3)$$
$$= \left(-4/3, 1/3, -7/3\right) = 1/3\,(-4,1,-7)$$

$$\|\vec{h}\| = \frac{1}{3}\sqrt{(-4)^2+(1)^2+(-7)^2} = \frac{1}{3}\sqrt{66}$$

$$\text{Area} = \frac{1}{2}\|\vec{h}\|\,\|\vec{AB}\| = \frac{1}{2}\left(\frac{\sqrt{66}}{3}\right)\sqrt{6}$$
$$= \frac{\sqrt{66}\sqrt{6}}{6} = \frac{\sqrt{66}}{\sqrt{6}} = \boxed{\sqrt{11}}$$

(ii) $\text{Area} = \frac{1}{2}\|\vec{AC} \times \vec{AB}\|$
$$= \frac{1}{2}\left\|\begin{pmatrix}-2\\0\\-2\end{pmatrix} \times \begin{pmatrix}-2\\-1\\1\end{pmatrix}\right\| = \frac{1}{2}\|(-2,6,2)\| = \frac{1}{2}\sqrt{44}$$