LAST NAME: SOLUTIONS

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QUIZ 3 (B)

DAWSON COLLEGE

201-NYC-05-S2 Linear Algebra

Instructor: E. Richer

Date: July 10th 2008

Question 1. *(10 marks)*
Let $\vec{u} = (1,2,-1)$, $\vec{v} = (1,1,-1)$ and $\vec{w} = (0,1,1)$.
Compute the following:
(i) $||2\vec{w}||$
(ii) $\text{proj}_{\vec{u}}\vec{v}$
(iii) $(\vec{w} \times \vec{v}) \times \vec{u}$
(iv) $(2\vec{w}) \cdot \vec{u}$
(v) The volume of the parrallelepiped defined by the vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$

(i) $\|2\vec{w}\| = 2\|\vec{w}\| = 2\sqrt{1^2+1^2} = \boxed{2\sqrt{2}}$

(ii) $\text{proj}_{\vec{u}}\vec{v} = \left(\dfrac{\vec{u}\cdot\vec{v}}{\vec{u}\cdot\vec{u}}\right)\vec{u} = \dfrac{4}{6}(1,2,-1) = \boxed{\left(2/3, 4/3, -2/3\right)}$

(iii) $(\vec{w} \times \vec{v}) \times \vec{u} = (-2,1,-1) \times \vec{u} = \boxed{(1,-3,-5)}$

$\begin{pmatrix}0\\1\\1\end{pmatrix} \begin{pmatrix}1\\1\\-1\end{pmatrix} \qquad \begin{pmatrix}-2\\1\\-1\end{pmatrix} \begin{pmatrix}1\\2\\-1\end{pmatrix}$

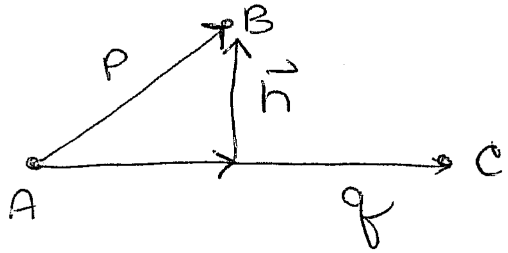
(iv) $(2\vec{w})\cdot\vec{u} = (0,2,2)\cdot(1,2,-1) = \boxed{2}$

(v) $V = |\vec{u}\cdot(\vec{v}\times\vec{w})| = |\vec{u}\cdot(2,-1,1)|$

$\begin{pmatrix}1\\1\\-1\end{pmatrix} \begin{pmatrix}0\\1\\1\end{pmatrix}$

$= |\ -1\ | = \boxed{1}$

Question 2. *(10 marks)*
A triangle is defined by the points $A(1,1,1)$, $B(2,0,-1)$ and $C(1,-1,1)$.
Compute the **area of the triangle** in two different ways:
(i) Using projections
(ii) Using the cross product



$$\vec{h} = \vec{p} - \text{proj}_{\vec{q}}\vec{p}$$

where $\vec{p} = \vec{AB} = (1,-1,-2)$
$\vec{q} = \vec{AC} = (0,-2,0)$

$$\vec{h} = \vec{p} - \frac{\vec{p}\cdot\vec{q}}{\vec{q}\cdot\vec{q}}\vec{q}$$
$$= (1,-1,-2) - \left(\frac{2}{4}\right)(0,-2,0)$$
$$= (1,-1,-2) - (0,-1,0) = (1,0,-2)$$

$$\text{AreA} = \frac{\|\vec{h}\|\|\vec{q}\|}{2}$$
$$= \frac{\sqrt{1^2+(-2)^2}\sqrt{(-2)^2}}{2} = \frac{\sqrt{5}(2)}{(2)}$$
$$\boxed{=\sqrt{5}}$$

(ii) $A = \frac{1}{2}\|\vec{p}\times\vec{q}\|$
$$= \frac{1}{2}\left\|\begin{pmatrix}1\\-1\\-2\end{pmatrix}\times\begin{pmatrix}0\\-2\\0\end{pmatrix}\right\| = \|(-4,0,-2)\|$$
$$= \frac{1}{2}\sqrt{20} = \boxed{\sqrt{5}}$$