LAST NAME: SOLUTIONS

FIRST NAME: ____________

STUDENT NUMBER: ____________

TEST 1 (A)

DAWSON COLLEGE

201-NYC-05-S2 - Linear Algebra

Instructor: E. Richer

Date: June 19th 2008

Question 1.

(a) *(4 marks)*

Define briefly the four conditions that a matrix must satisfy to be in reduced row echelon form.

(1) Any rows with all zero entries at the bottom

(2) First non zero entry in any row is a leading 1

(3) All leading 1's are to the right of each other (from top row to bottom)

(4) Each leading 1 has zeros above & below it

(b) *(6 marks)*

Using elementary row operations, find the **reduced row echelon** form of the following matrix. Show each step, writing a new matrix showing the result of each of your elementary row operations.

$$A = \begin{bmatrix} 0 & 1 & 1 & 4 \\ 1 & 1 & 2 & -1 \\ 3 & 4 & 7 & 1 \end{bmatrix} \quad R_1 \leftrightarrow R_2 \quad \begin{bmatrix} 1 & 1 & 2 & -1 \\ 0 & 1 & 1 & 4 \\ 3 & 4 & 7 & 1 \end{bmatrix} \quad R_3 - 3R_1 \rightarrow R_3 \quad \begin{bmatrix} 1 & 1 & 2 & -1 \\ 0 & 1 & 1 & 4 \\ 0 & 1 & 1 & 4 \end{bmatrix}$$

$$R_3 - R_2 \rightarrow R_3 \quad \begin{bmatrix} 1 & 1 & 2 & -1 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_1 - R_2 \rightarrow R_1 \quad \begin{bmatrix} 1 & 0 & 1 & -5 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Question 2. *(10 marks)*
Consider the system whose augmented matrix is given in row echelon form below, where a and b are constants:

$$\begin{bmatrix} 1 & 0 & 3 & 5 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & a^2-1 & b-3 \end{bmatrix}$$

State the conditions on a and b so that the corresponding linear system has:
(i) No solutions
(ii) Infinitely many solutions
(iii) A unique solution

(i) $a^2-1=0$ & $b-3 \neq 0$

$\boxed{a = \pm 1}$ $\boxed{b \neq 3}$

(ii) $a^2-1=0$

$\boxed{a=\pm 1}$ & $b-3=0$

$\boxed{b=3}$

(iii) $a^2-1 \neq 0$

$\boxed{a \neq \pm 1}$

Question 3. *(10 marks)*
Solve the following system of equations (note that there are 4 variables).
$x_1 + 2x_2 - x_3 = 1$
$2x_1 + 4x_2 - 2x_4 = 6$
$-2x_1 - 4x_2 + 2x_3 = -2$

$$\begin{bmatrix} 1 & 2 & -1 & 0 & 1 \\ 2 & 4 & 0 & -2 & 6 \\ -2 & -4 & 2 & 0 & -2 \end{bmatrix} \quad \begin{array}{l} R_2 - 2R_1 \to R_2 \\ R_3 + R_1 \to R_1 \end{array} \quad \begin{bmatrix} 1 & 2 & -1 & 0 & 1 \\ 0 & 0 & 2 & -2 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\tfrac{1}{2} R_2 \to R_2 \quad \begin{bmatrix} 1 & 2 & -1 & 0 & 1 \\ 0 & 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

X_2, X_4 ARE FREE

let $X_2 = s$
$X_4 = t$

$X_3 - X_4 = 2$
$X_3 = 2 + X_4$
$= 2 + t$

$X_1 + 2X_2 - X_3 = 1$
$X_1 = 1 - 2X_2 + X_3$
$= 1 - 2s + 2 + t$
$= 3 - 2s + t$

$(X_1, X_2, X_3, X_4) = (3 - 2s + t, s, 2 + t, t)$
s, t in $\mathbb{R}$

Question 4. *(10 marks)*

$$A = \begin{bmatrix} 2 & -4 \\ 3 & 1 \\ -1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 1 & -3 \\ 1 & 0 & -2 \end{bmatrix} \quad C = \begin{bmatrix} 1 & -2 \\ 3 & 2 \end{bmatrix}$$

Compute the following (where possible).

(a) CA

(b) $A + B^T$

(c) $CB - 2A^T$

(a) $\underset{2\times2}{C}\,\underset{3\times2}{A}$ Not defined

(b) $A + B^T$

$$= \begin{bmatrix} 2 & -4 \\ 3 & 1 \\ -1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ -3 & -2 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ 4 & 1 \\ -4 & -2 \end{bmatrix}$$

(c) $CB - 2A^T$

$$\begin{bmatrix} 1 & -2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 0 & 1 & -3 \\ 1 & 0 & -2 \end{bmatrix} - 2\begin{bmatrix} 2 & 3 & -1 \\ -4 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 1 & 1 \\ 2 & 3 & -13 \end{bmatrix} - \begin{bmatrix} 4 & 6 & -2 \\ -8 & 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & -5 & 3 \\ 10 & 1 & -13 \end{bmatrix}$$

Question 5. *(10 marks)*
Find A given the information below.

$$(2A^T)^{-1} = \begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$$

$$\left((2A^T)^{-1}\right)^{-1} = \begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}^{-1}$$

$$2A^T = \frac{1}{10}\begin{bmatrix} 2 & 1 \\ -4 & 3 \end{bmatrix}$$

$$A^T = \frac{1}{20}\begin{bmatrix} 2 & 1 \\ -4 & 3 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1/10 & 1/20 \\ -1/5 & 3/20 \end{bmatrix}$$

$$A = \begin{bmatrix} 1/10 & -1/5 \\ 1/20 & 3/20 \end{bmatrix}$$

Question 6. *(10 marks)*
Given n x n invertible matrices A and B. State whether or not the following statements are TRUE or FALSE. If the statement is false give an example otherwise simply state that the statement is true.
(a) $A+B$ is invertible.

FALSE

$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

A, B invertible but

$A+B = 0$ Not invertible

(b) $(AB)^T = B^T A^T$

TRUE

(c) $(AB)^{-1} = B^{-1}A^{-1}$

TRUE

Question 7.

(a) *(3 marks)*

Given a square matrix A, state the two equations that a matrix C must satisfy in order for $C = A^{-1}$.

(b) *(6 marks)*

Let A and B be square matrices satisfying $AB = 0$. Show that if A is invertible then $B = 0$

(a) $AC = I$

$CA = I$

(b) $AB = O$

IF A is invertible then it has An inverse A^{-1} satisfying

$AA^{-1} = I$

$A^{-1}A = I$

so $AB = O$ multiply both sides by A^{-1}

$A^{-1}AB = A^{-1}O$

$IB = O$

$B = O$ $\blacksquare$