15 February 2006 **MATH 133 Test** Time: 6pm-8pm

Instructions: Show all necessary steps and details in your work. Simplify answers as far as possible. There are 6 questions on the two sides of this test paper. Answer all questions in the exam booklets. Calculators and notes are not permitted. Keep this test paper when finished.

1. (10 points) Obtain all the values of a and b for which the system

   $2x_1 - x_2 + x_3 = 6$

   $x_1 + 2x_2 - ax_3 = 11$

   $4x_1 + 3x_2 - x_3 = b$

   will be

   (i) *Consistent Determinate,* i.e. have a unique solution.

   (ii) *Consistent Indeterminate,* i.e. have an infinite number of solutions.

   (iii) *Inconsistent,* i.e. have no solution.

2. (10 points) Let $A = \begin{bmatrix} 0 & 9 \\ 5 & -4 \end{bmatrix}$.

   (a) Write A^{-1} as a product of elementary matrices.

   (b) Write A as a product of elementary matrices.

3. (5 points)

   (a) Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ and $B = \begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix}$. Write out the product AB.

   (b) Find all 2×2 matrices B of the form $B = \begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix}$ such that $B^2 = B$. (Hint: There are four such B.)

continued on reverse ...

4. (5 points)

   (a) Given a square matrix A, what two equations must a matrix C satisfy in order that $C = A^{-1}$? (i.e. State the definition that identifies C as the inverse of A.)

   (b) Use the above definition to prove that $(AB)^{-1} = B^{-1}A^{-1}$ if A and B are invertible $n \times n$ matrices.

5. (5 points) Given the determinant $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 5$, evaluate $\begin{vmatrix} g & h & i \\ 7a-3g & 7b-3h & 7c-3i \\ 2d & 2e & 2f \end{vmatrix}$.

6. (5 points) If A, B and C are 3×3 matrices with $\det A = -5$, $\det B = 3$, and $\det C = 7$, evaluate

   (a) $\det(\ 2B^T A^{-1} B^{-1} C^2 A C^{-1}\ )$

   (b) $\det(\ \mathrm{adj}(A)\ )$.