

**Instructions:** THIS QUESTION SHEET CANNOT BE TAKEN FROM THE EXAM ROOM. ANSWER ALL QUESTIONS IN THE EXAM BOOKLETS. NO CALCULATORS, CELL-PHONES, OR OTHER ELECTRONIC DEVICES ALLOWED. NO BOOKS OR NOTES ALLOWED.

1. (5 points) Let  $P = (1, 2, 3)$  and  $Q = (7, 14, 21)$ . Let  $M$  be the midpoint of the line segment  $\overline{PQ}$  and  $R$  and  $S$  be the points that divide  $\overline{PM}$  (**not**  $\overline{PQ}$ ) into three equal parts. Sketch the situation and find  $M$ ,  $R$  and  $S$ .

2. (5 points)

- (a) Define briefly the three types of elementary row operations on a matrix.
- (b) Using elementary row operations, find the **reduced** echelon form of the following matrix. Show each step - write a new matrix showing the result of each of your elementary row operations.

$$\begin{bmatrix} 0 & 1 & 1 & 4 \\ 1 & 1 & 2 & -1 \\ 3 & 4 & 7 & 1 \end{bmatrix}$$

3. (5 points)

- (a) Evaluate each of the following matrix products:

$$C = \begin{bmatrix} 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = ? \quad , \quad D = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 4 & 5 & 6 \end{bmatrix} = ?$$

- (b) Find  $\text{rank}(C)$  and  $\text{rank}(D)$ , justifying your answers.

4. (10 points) Let  $A = \begin{bmatrix} 1 & -2 & 2 & 2 & 0 \\ -3 & 6 & -6 & -6 & 0 \\ -1 & 2 & 2 & 6 & 4 \\ 1 & -2 & 3 & 4 & 1 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & -2 & 0 & -2 & -2 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ .

Given that  $B$  is the reduced echelon form of  $A$ , answer the following:

- (a) Find the dimension of the row space of  $A$  and a basis of it.
- (b) Find the dimension of the column space of  $A$  and a basis of it.
- (c) Find the dimension of the null space of  $A$  and a basis of it.

- (d) Is it true or false that  $\text{col}(A) = \text{Span} \left\{ \begin{bmatrix} -2 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$ ? Why do you say so?