

Test 3

This test is graded out of 63 marks. No books, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (2 marks)

Express the percent 0.2% as a fraction and as a decimal.

$$0.2\% = 0.002, \quad 0.2\% = \frac{2}{1000} = \frac{1}{500}$$

Question 2. (4 marks)

Evaluate the following to two decimal places:

1. $\sqrt{4638} = 3.34$
2. $(74620)^{\frac{2}{3}} = 2.34 \times 10^{11}$
3. $-(2)^{-1} = -0.5$
4. $(2.78)^1 = 2.78$

Question 3. (3 marks)

Alexander invests \$5 300 in a bank account for 3 years and 2 months at 9% interest p.a. compounded monthly. What is the future value of Alexander's investment? How much interest does Alexander earn?

$$m = 12$$

$$i = \frac{j}{m} = \frac{9\%}{12} = 0.0075$$

$$n = \left(3 \frac{2}{12}\right) 12 = 38$$

$$FV = PV(1+i)^n$$

$$FV = 5300(1+0.0075)^{38}$$

$$= 7040.25$$

∴ the future value of his investment is \$7040.25

$$\text{Interest} = FV - PV$$

$$= 7040.25 - 5300$$

$$= 1740.25$$

∴ the interest earned is \$1740.25.

Question 4. (6 marks)

Henri wants to invest \$55 000 in a savings bank account. He has two choices: Bank Double-Interest offers a savings account with an interest rate of 3.75% compounded monthly, Cookie Bank offers a savings account with interest rate of 4% compounded yearly. Suppose the investment period is 6 years and 6 months. Which bank should Henri choose? What is the difference in the amount of interest earned between the two banks?

Cookie Bank:

$$\begin{aligned} FV &= PV(1+i)^n \\ &= 55000(1+0.04)^{6.5} \\ &= \$70\,970.75 \end{aligned}$$

Bank Double-Interest:

$$\begin{aligned} FV &= PV(1+i) \\ &= 55000\left(1 + \frac{0.0375}{12}\right)^{12 \cdot 6.5} \\ &= \$70\,154.72 \end{aligned}$$

∴ the best choice is the Cookie Bank

The difference in interest is

$$70\,970.75 - 70\,154.72 = \$816.03$$

∴ the difference is \$816.03

Question 5. (6 marks)

Marc wins the lottery and has the option of getting \$1 000 000 now or \$500 000 now and \$600 000 in 10 years. Since Marc was a former AEC student he chose the right option. If money was worth 3.75% compounded monthly what did did Mark choose? By how much was his choice better in present value money?

option 1:

\$1 000 000

option 2:

In present value:

$$500\,000 + \frac{FV}{(1+i)^n}$$

$$= 500\,000 + \frac{600\,000}{(1+0.003125)^{120}}$$

$$= 500\,000 + 412\,614.76$$

$$= \$912\,614.76$$

$$FV = \$600\,000, m = 12$$

$$i = \frac{j}{m} = \frac{0.0375}{12} = 0.003125$$

$$n = 10m = 10(12) = 120$$

∴ option 1 is best and the difference is $1\,000\,000 - 912\,614.76 = \$87\,385.24$

Question 6. (3 marks)

What sum of money will accumulate to \$2 600 over 2 years and 3 months if the money is worth 4.75% compounded quarterly?

$$P = \frac{S}{(1+i)^n}$$

$$= \frac{2600}{(1+0.011875)^9}$$

$$= \$2\,337.93$$

$$m = 4$$

$$n = (2\frac{3}{12}) \cdot 4 = 9$$

$$i = \frac{j}{m} = \frac{0.0475}{4} = 0.011875$$

$$S = \$2\,600$$

∴ the sum of money needed is \$2 337.93

Question 7. (3 marks)

What effective rate is equivalent to a nominal rate of 4.75% compounded quarterly?

$$f = (1+i)^m - 1 \quad \text{where } i = \frac{j}{m} = \frac{0.0475}{4} = 0.011875$$

$$= (1+0.011875)^4 - 1$$

$$= 4.84\%$$

$m = 4$

∴ the equivalent effective rate is 4.84%

Question 8. (3 marks)

Joe wants to invest in a savings account, he deposits \$1 in his savings account every day for 10 years. What will be the balance of his savings account in 10 years if his savings account has a nominal interest rate of 3.75% compounded daily?

$$FV = PMT \left[\frac{(1+i)^n - 1}{i} \right]$$

$$= 1 \cdot \left[\frac{\left(1 + \frac{0.0375}{365}\right)^{3650} - 1}{\frac{0.0375}{365}} \right]$$

$$= \$4\,428.31$$

$$m = 365$$

$$i = \frac{j}{m} = \frac{0.0375}{365}$$

$$PMT = 1$$

$$n = 10 \cdot 365 = 3650$$

∴ the balance of his savings account will be \$4 428.31.

Question 9. (3 marks)

Yann got a loan of \$3 200 to pay for a super fast computer. He is to repay the loan by payments of \$100 monthly. If the interest is 8.25% compounded monthly, how many months will it take Yann to repay the loan?

where $PMT = 100$, $PV = 3200$, $i = \frac{0.0825}{12} = 0.006875$.

$$PV = PMT \left[\frac{1 - (1+i)^{-n}}{i} \right]$$

$$3200 = 100 \left[\frac{1 - (1.006875)^{-n}}{0.006875} \right]$$

$$0.22 = 1 - (1.006875)^{-n}$$

$$(1.006875)^{-n} = 1 - 0.22$$

$$\begin{aligned} \ln(1.006875)^{-n} &= \ln 0.78 \\ -n \ln(1.006875) &= \ln 0.78 \\ n &= \frac{-\ln 0.78}{\ln 1.006875} \\ &= 36.26 \end{aligned}$$

∴ it will take Yann 37 months to repay the computer

Question 10. (3 marks)

George wants to receive \$500 at the end of every month for two years, how much does he need to deposit at the beginning of the two-year period if the interest rate is 6% compounded monthly?

$$PV = PMT \left[\frac{1 - (1+i)^{-n}}{i} \right]$$

$$= 500 \left[\frac{1 - (1+0.005)^{-24}}{0.005} \right]$$

$$= \$11\,281.43$$

$$\begin{aligned} m &= 12 \\ i &= \frac{j}{m} = \frac{0.06}{12} = 0.005 \\ n &= 2 \cdot m = 24 \end{aligned}$$

∴ he needs \$11 281.43.

Question 11. (3 marks)

What deposit made at the end of each quarter for 10 years will accumulate to \$15 000 at an interest rate of 6% compounded quarterly?

$$FV = PMT \left[\frac{(1+i)^n - 1}{i} \right]$$

$$PMT = \frac{FV}{\left[\frac{(1+i)^n - 1}{i} \right]}$$

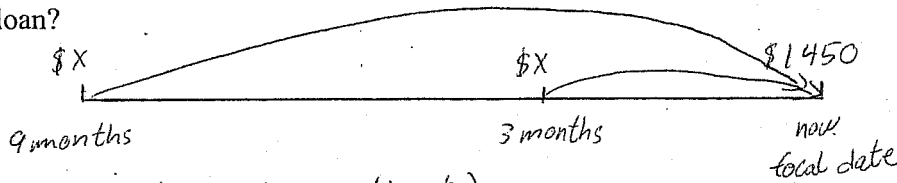
$$PMT = \frac{15\,000}{\left[\frac{(1+\frac{0.06}{4})^{40} - 1}{\frac{0.06}{4}} \right]}$$

$$= \$276.41$$

∴ payments of \$276.41 are needed.

Question 12. (5 marks)

Darcy took two equal sized loans, one 3 months ago and the other 9 months ago. He repaid the loans today, a total of \$1 450. If the interest on the loans was 9% p.a. then what was the size of each loan?



$$1450 = x(1+rt_1) + x(1+rt_2)$$

$$1450 = x(1+0.09(\frac{9}{12})) + x(1+0.09(\frac{3}{12}))$$

$$1450 = 1.0675x + 1.0225x$$

$$1450 = 2.09x$$

$$\$ 693.78 = x$$

∴ the size of each loan was \$693.78

Question 13. (4 marks)

The component cost to make an OGG Vorbis player is four-ninth of the total cost and labour is one-third of the component cost. If cost of labour is \$10 what is the total cost of the MP3/OGG Vorbis player.

$$\frac{\text{Component Cost}}{\text{Total Cost}} = \frac{4}{9}$$

$$\frac{\text{Labour Cost}}{\text{Component Cost}} = \frac{1}{3}$$

$$\Rightarrow \text{Component Cost} = 3(\text{Labour Cost}) = 30$$

$$\begin{aligned} \text{Total Cost} &= \frac{9}{4}(\text{Component Cost}) \\ &= \frac{9(30)}{4} \\ &= 67.5 \end{aligned}$$

∴ the total cost is \$67.50

Question 14. (5 marks)

A store buys a sofa for \$1 200 less a trade discount of 25%, 15%, 5%. What is the selling price of the sofa if the store expenses are 15% of the selling price and require a profit of 20% on the selling price. What is the selling price? What is the markup based on the cost?

$$\begin{aligned} \text{Cost} &= (\text{List price})(1-d_1)(1-d_2)(1-d_3) \\ \text{Cost} &= (1200)(1-0.25)(1-0.15)(1-0.05) \\ \text{Cost} &= \$726.75 \end{aligned}$$

$$\begin{aligned} \text{Selling Price} &= \text{Cost} + \text{Expense} + \text{profit} \\ S &= 726.75 + 15\% \text{ of } S + 20\% \text{ of } S \\ S &= 726.75 + 0.15S + 0.20S \end{aligned}$$

$$\begin{aligned} 0.65S &= 726.75 \\ S &= \$1118.38 \end{aligned}$$

∴ the selling price is \$1118.38

$$\therefore \text{rate of markup w.r.t. cost} = \frac{1118.38 - 726.75}{726.75} = 53.84\%$$

Question 16. (3 marks)

Bobby received \$44.01 interest for a deposit of \$5 100 invested for a period of 90 days. What was the rate of interest p.a.?

$$I = Prt$$

$$r = \frac{I}{Pt}$$

$$= \frac{44.01}{5100 \left(\frac{90}{365}\right)} = 3.5\%$$

∴ the rate of interest is 3.5%

Question 17. (2 marks)

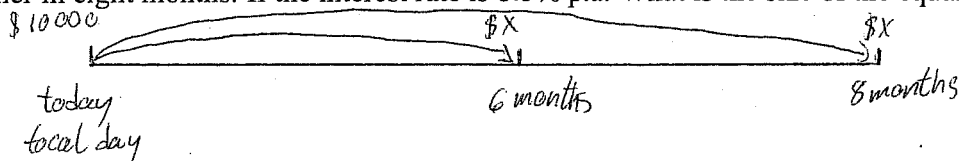
Compute the future value of \$1 423 over seven months at $2\frac{1}{4}\%$ p.a.?

$$\begin{aligned} S &= P(1+rt) \\ &= 1423\left(1 + 0.0225\left(\frac{7}{12}\right)\right) \\ &= \$1441.68 \end{aligned}$$

∴ the future value is \$1441.68

Question 18. (5 marks)

Ron borrowed \$10 000 today and is to repay the loan in two equal payments, one in six months and the other in eight months. If the interest rate is 8.5% p.a. What is the size of the equal payments?



$$10000 = \frac{X}{(1+rt_1)} + \frac{X}{(1+rt_2)}$$

$$10000 = \frac{X}{(1+0.085\left(\frac{6}{12}\right))} + \frac{X}{(1+0.085\left(\frac{8}{12}\right))}$$

$$10000 = 0.959232613X + 0.946372239X$$

$$X = \$5247.68$$

∴ the payments are each of \$5247.68

Bonus Question (3 marks)

Derive the formula for future value of a simple ordinary annuity from a geometric progression.

FV