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## Test 3

This test is graded out of 63 marks. No books, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

**Question 1.** (2 marks)

Express the percent 0.3% as a fraction and as a decimal.

$$\frac{3}{1000}, 0.003$$

**Question 2.** (4 marks)

Evaluate the following to two decimal places:

1.  $\sqrt{4523} = 3.33$
2.  $(34643)^{\frac{1}{3}} = 36.81508632$
3.  $-(3)^{-1} = -\frac{1}{3} = 0.33$
4.  $(2.89)^1 = 2.89$

**Question 3.** (3 marks)

Alexander invests \$6 500 in a bank account for 2 years and 5 months at 7% interest p.a. compounded monthly. What is the future value of Alexander's investment? How much interest does Alexander earn?

$$m = 12$$

$$i = \frac{j}{m} = \frac{7\%}{12} = \frac{0.07}{12}$$

$$n = 12 \left( 2 \frac{5}{12} \right) = 29$$

$$\begin{aligned} FV &= PV(1+i)^n \\ &= 6500 \left( 1 + \frac{0.07}{12} \right)^{29} \\ &= \$7694.28 \end{aligned}$$

$$\begin{aligned} \text{Interest} &= FV - PV \\ &= 7694.28 - 6500 \\ &= \$1194.28 \end{aligned}$$

$\therefore$  the future value of Alexander is \$7694.28 and he earned 1194.28 interest.

**Question 4. (6 marks)**

Henri wants to invest \$65 000 in a savings bank account. He has two choices: Bank Double-Interest offers a savings account with an interest rate of 2.75% compounded monthly, Cookie Bank offers a savings account with interest rate of 3% compounded yearly. Suppose the investment period is 6 years and 6 months. Which bank should Henri choose? What is the difference in the amount of interest earned between the two banks?

Bank Double-Interest: 
$$FV = PV(1+i)^n$$
$$= 65000 \left(1 + \frac{0.0275}{12}\right)^{12 \cdot 6.5}$$
$$= \$77\,706.02$$

Cookie Bank: 
$$FV = PV(1+i)^n$$
$$= 65000(1+0.03)^{6.5}$$
$$= \$78\,769.00$$

∴ Henri should choose the Cookie Bank

The interest difference is  
 $78\,769.00 - 77\,706.02$   
 $= 1062.98$   
∴ the difference is 1062.98

**Question 5. (6 marks)**

Marc wins the lottery and has the option of getting \$2 000 000 now or \$1 000 000 now and \$1 200 000 in 15 years. Since Mark was a former AEC student he chose the right option. If money was worth 4.75% compounded monthly what did did Mark choose? By how much was his choice better in present value money?

Option 1:  
\$ 2 000 000

Option 2: In present value:

$$1\,000\,000 + \frac{FV}{(1+i)^n}$$
$$= 1\,000\,000 + \frac{1\,200\,000}{\left(1 + \frac{0.0475}{12}\right)^{180}}$$
$$= 1\,000\,000 + 589\,328.22$$
$$= \$1\,589\,328.22$$

where:  $FV = 1\,200\,000$   
 $m = 12$   
 $i = \frac{j}{m} = \frac{0.0475}{12}$   
 $n = 12(15) = 180$

∴ he should choose the first option.

The difference between the two  $2\,000\,000 - 1\,589\,328.22$   
 $= \$410\,671.78$

**Question 6. (3 marks)**

What sum of money will accumulate to \$5 300 over 4 years and 6 months if the money is worth 5.75% compounded quarterly?

$$P = \frac{S}{(1+i)^n}$$

$$= \frac{5300}{(1+0.014375)^{18}}$$

$$= \$4099.23$$

$$m = 4$$

$$n = \left(4\frac{6}{12}\right)4 = 18$$

$$i = \frac{j}{m} = \frac{0.0575}{4} = 0.014375$$

$$S = \$5300$$

∴ the sum of money needed is \$4099.23

**Question 7. (3 marks)**

What effective rate is equivalent to a nominal rate of 3.75% compounded quarterly?

$$f = (1+i)^m - 1$$

$$= (1+0.009375)^4 - 1$$

$$= 3.80\%$$

where  $m = 4$

$$i = \frac{j}{m} = \frac{0.0375}{4} = 0.009375$$

∴ the equivalent effective rate is 3.80%

**Question 8. (3 marks)**

Joe wants to invest in a savings account, he deposits \$2 in his savings account every day for 20 years. What will be the balance of his savings account in 20 years if his savings account has a nominal interest rate of 2.75% compounded daily?

$$FV = PMT \left[ \frac{(1+i)^n - 1}{i} \right]$$

$$= 2 \left[ \frac{\left(1 + \frac{0.0275}{365}\right)^{7300} - 1}{\frac{0.0275}{365}} \right]$$

$$m = 365$$

$$i = \frac{j}{m} = \frac{0.0275}{365}$$

$$PMT = 2$$

$$n = 365(20) = 7300$$

$$= \$19\,463.58$$

∴ the balance of his account will be \$19 463.58.

**Question 9. (3 marks)**

Yann got a loan of \$4 100 to pay for a super fast computer. He is to repay the loan by payments of \$200 monthly. If the interest is 7.25% compounded monthly, how many months will it take Yann to repay the loan?

where  $PMT = 200$ ,  $PV = 4100$ ,  $m = 12$ ,  $i = \frac{0.075}{12} = 0.00625$

$$PV = PMT \left[ \frac{1 - (1+i)^{-n}}{i} \right]$$

$$4100 = 200 \left[ \frac{1 - (1 + 0.00625)^{-n}}{0.00625} \right]$$

$$0.128125 = 1 - (1 + 0.00625)^{-n}$$

$$\ln(1.00625)^{-n} = \ln(0.871875)$$

$$n = - \frac{\ln(0.871875)}{\ln(1.00625)}$$

$$n = 23$$

∴ It will take Yann 23 months to repay the loan.

**Question 10. (3 marks)**

George wants to receive \$700 at the end of every month for two years, how much does he need to deposit at the beginning of the two-year period if the interest rate is 5% compounded monthly?

$$PV = PMT \left[ \frac{1 - (1+i)^{-n}}{i} \right]$$

where  $PMT = 700$ ,  $m = 12$ ,  $i = \frac{0.05}{12}$ ,  $n = 12 \cdot 2 = 24$

$$= 700 \left[ \frac{1 - (1 + \frac{0.05}{12})^{-24}}{\frac{0.05}{12}} \right]$$

∴ he needs \$15 955.73

$$= \$15\,955.73$$

**Question 11. (3 marks)**

What deposit made at the end of each quarter for 15 years will accumulate to \$25 000 at an interest rate of 7% compounded quarterly?

$$FV = PMT \left[ \frac{(1+i)^n - 1}{i} \right]$$

$$PMT = \frac{FV}{\left[ \frac{(1+i)^n - 1}{i} \right]}$$

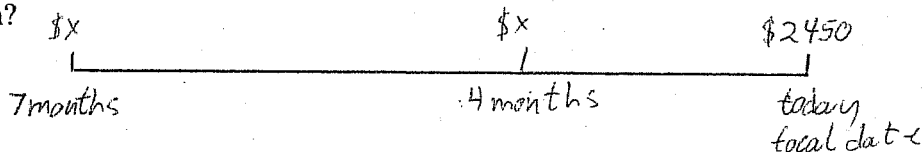
$$= \frac{25\,000}{\left[ \frac{(1 + \frac{0.07}{4})^{60} - 1}{\frac{0.07}{4}} \right]}$$

∴ the payment is \$238.83

$$= \$238.83$$

**Question 12. (5 marks)**

Darcy took two equal sized loans, one 4 months ago and the other 7 months ago. He repaid the loans today, a total of \$2 450. If the interest on the loans was 7% p.a. then what was the size of each loan?



$$2450 = x(1+rt_1) + x(1+rt_2)$$

$$2450 = x\left(1+0.07\left(\frac{4}{12}\right)\right) + x\left(1+0.07\left(\frac{7}{12}\right)\right)$$

$$2450 = 1.02\bar{3}x + 1.0408\bar{3}x$$

$$x = \$1186.92$$

∴ the size of the loan is \$1186.92.

**Question 13. (4 marks)**

The component cost to make an OGG Vorbis player is four-ninth of the total cost and labour is one-third of the component cost. If cost of labour is \$11 what is the total cost of the MP3/OGG Vorbis player.

$$\frac{\text{Component Cost}}{\text{Total Cost}} = \frac{4}{9}$$

$$\frac{\text{Labour Cost}}{\text{Component Cost}} = \frac{1}{3}$$

$$\begin{aligned} \text{Component Cost} &= 3(\text{Labour Cost}) \\ &= 3(11) = 33 \end{aligned}$$

$$\begin{aligned} \text{Total Cost} &= \frac{9}{4}(\text{Component Cost}) \\ &= \frac{9}{4}(33) \\ &= 74.25 \end{aligned}$$

∴ the Total Cost is \$74.25.

**Question 14. (5 marks)**

A store buys a sofa for \$2 500 less a trade discount of 35%, 25%, 15%. What is the selling price of the sofa if the store expenses are 25% of the selling price and require a profit of 15% on the selling price. What is the selling price? What is the markup based on the cost?

$$\begin{aligned} \text{Cost} &= (\text{List price})(1-d_1)(1-d_2)(1-d_3) \\ &= 2500(1-0.35)(1-0.25)(1-0.15) \\ &= \$1035.94 \end{aligned}$$

$$\text{Selling price} = \text{Cost} + \text{Expense} + \text{Profit}$$

$$S = C + E + P$$

$$S = 1035.94 + 25\% \text{ of } S + 15\% \text{ of } S$$

$$S = 1035.94 + 0.25S + 0.15S$$

$$0.6S = 1035.94$$

$$S = \$1726.57$$

The rate of markup

$$\text{based on cost } \frac{1726.57 - 1035.94}{1035.94}$$

$$= 66.67\%$$

∴ the selling price is \$1726.57

**Question 16. (3 marks)**

Bobby received \$55.01 interest for a deposit of \$6 200 invested for a period of 120 days. What was the rate of interest p.a.?

$$I = Pt$$

$$r = \frac{I}{Pt}$$

$$= \frac{55.01}{6200 \left( \frac{120}{365} \right)}$$

$$= 2.70\%$$

∴ the interest rate is 2.70%

**Question 17. (2 marks)**

Compute the future value of \$2 123 over seven months at  $3\frac{3}{4}\%$  p.a.?

$$S = P(1 + rt)$$

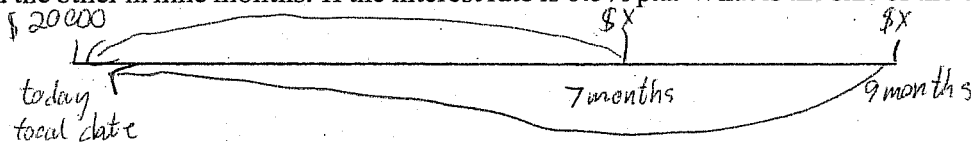
$$= 2123 \left( 1 + 0.0375 \left( \frac{7}{12} \right) \right)$$

$$= \$2169.44$$

∴ the future value is 2169.44

**Question 18. (5 marks)**

Ron borrowed \$20 000 today and is to repay the loan in two equal payments, one in seven months and the other in nine months. If the interest rate is 6.5% p.a. What is the size of the equal payments?



$$20000 = \frac{X}{(1+rt_1)} + \frac{X}{(1+rt_2)}$$

$$20000 = \frac{X}{\left(1 + 0.065 \left(\frac{7}{12}\right)\right)} + \frac{X}{\left(1 + 0.065 \left(\frac{9}{12}\right)\right)}$$

$$20000 = 0.963468486 X + 0.95351609 X$$

$$X = \$10433.05$$

∴ the size of the equal payment is \$10 433.05

**Bonus Question (3 marks)**

Derive the formula for future value of a simple ordinary annuity from a geometric progression.