Example:

$$\sum_{n=0}^{\infty} \frac{5}{3^n} = \sum_{n=0}^{\infty} 5\left(\frac{1}{3}\right)^n$$

$$= 5\left(\frac{1}{3}\right)^0 + 5\left(\frac{1}{3}\right)^1 + 5\left(\frac{1}{3}\right)^2 + 5\left(\frac{1}{3}\right)^3 + 5\left(\frac{1}{3}\right)^4 + \dots + 5\left(\frac{1}{3}\right)^n + \dots$$

$$= 5 + \frac{5}{3} + \frac{5}{9} + \frac{5}{27} + \frac{5}{81} + \dots + 5\left(\frac{1}{3}\right)^n + \dots$$

$$= \frac{a}{1-r} = \frac{5}{1-\frac{1}{3}} = \frac{5}{2/3} = \frac{15}{2}$$

$$\sum_{n=\cancel{0}}^{\infty} 7\left(\frac{1}{5}\right)^n = \sum_{n=1}^{\infty} 7\left(\frac{1}{5}\right)^n + a_0 - a_0$$

add and subtract the zero term.

notice $n=1$ not $n=0$

$$= \sum_{n=0}^{\infty} 7\left(\frac{1}{5}\right)^n - a_0$$

$$= \frac{7}{1-\frac{1}{5}} - 7\left(\frac{1}{5}\right)^0$$

$$= \frac{7}{\frac{4}{5}} - 7$$

$$= \frac{35}{4} - 7 = \frac{35-28}{4} = \frac{7}{4}$$

Example:

$$\sum_{n=1}^{\infty}\left[\frac{1}{2^{n+1}}-\frac{1}{5^{n-1}}\right]$$

Lets look at both infinite series independently.

$$\sum_{n=1}^{\infty}\frac{1}{2^{n+1}} = \sum_{n=1}^{\infty}\frac{1}{2\cdot 2^{n}}$$

$$= \sum_{n=1}^{\infty}\frac{1}{2}\left(\frac{1}{2}\right)^{n}$$

$$= \sum_{n=1}^{\infty}\frac{1}{2}\left(\frac{1}{2}\right)^{n} + a_0 - a_0$$

add and remove the zero term.

$$= \sum_{n=0}^{\infty}\frac{1}{2}\left(\frac{1}{2}\right)^{n} - a_0$$

$$= \frac{1/2}{1-1/2} - \frac{1}{2}\left(\frac{1}{2}\right)^{0} = \frac{1/2}{1/2} - \frac{1}{2} = \frac{2}{2} - \frac{1}{2} = \frac{1}{2}$$

$$\sum_{n=1}^{\infty}\frac{1}{5^{n-1}} = \sum_{n=1}^{\infty}\frac{5}{5^{n}}$$

$$= \sum_{n=1}^{\infty}5\left(\frac{1}{5}\right)^{n}$$

$$= \sum_{n=1}^{\infty}5\left(\frac{1}{5}\right)^{n} + a_0 - a_0$$

add and remove the zero term

$$= \sum_{n=0}^{\infty}5\left(\frac{1}{5}\right)^{n} - a_0$$

$$= \frac{5}{1-\frac{1}{5}} - 5\left(\frac{1}{5}\right)^{0}$$

$$= \frac{5}{4/5} - 5 = \frac{25}{4} - \frac{20}{4} = \frac{5}{4}$$

Since both infinite series converge

$$\sum_{n=1}^{\infty}\left[\frac{1}{2^{n+1}}-\frac{1}{5^{n-1}}\right]=\sum_{n=1}^{\infty}\frac{1}{2^{n+1}}-\sum_{n=1}^{\infty}\frac{1}{5^{n-1}}$$

$$=\frac{1}{2}-\frac{5}{4}$$

$$=\frac{2}{4}-\frac{5}{4}$$

$$=-\frac{3}{4}$$