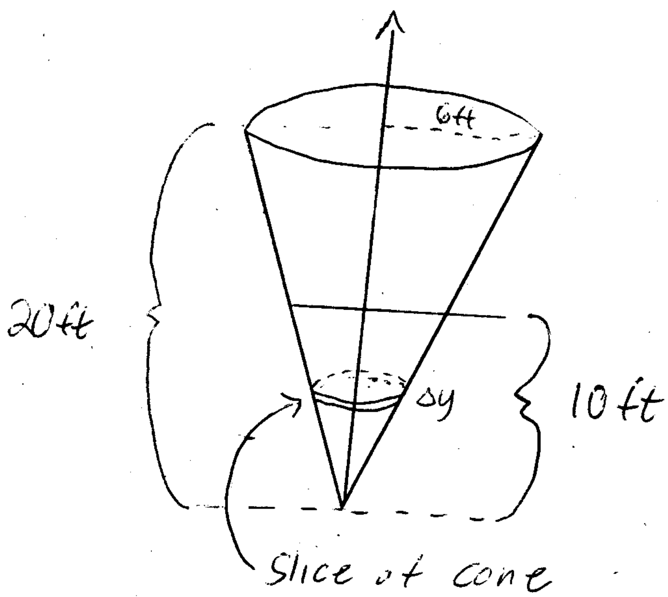
Example:

A conical tank 20 ft high with circular top 12 ft in diameter with its tip pointing down. Find the work done by filling half (in heigth) the tank from a source 10 ft below the bottom of the tank. (The water weighs 62.4 pounds per cubic foot.)

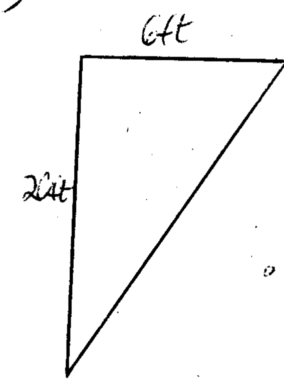
First draw a diagram:




Volume of Slice:

$$\Delta V = \pi x^2 \Delta y$$

but we want the volume of the slice with respect to the y-axis. We need to find a relation between x and y.




$$\therefore \frac{6}{20} = \frac{x}{y}$$

$$\frac{6y}{20} = x$$

$$\frac{3y}{10} = x$$

$\therefore$ the volume of the slice becomes

$$\Delta V = \pi \left(\frac{3y}{10}\right)^2 \Delta y = \frac{\pi 9}{100} y^2 \Delta y$$

## Force of Slice:

$$\Delta F = (\text{Weighs per cubic foot of liquid})(\Delta V)$$

$$\Delta F = \frac{62.4(\pi)(9)}{100}\, y^2 \Delta y$$

## Work of Slice:

$$\Delta W = \Delta F d$$

$$\Delta W = \frac{62.4(\pi)(9)}{100}\, y^2 (y+10)\, \Delta y$$

since the tank is located 10ft below the tank.

## Setting the integral:

$$W = \int_0^{10} \frac{62.4(\pi)(9)}{100}\, y^2 (y+10)\, dy$$

notice the $\Delta y$ changes to $dy$

since we are filling the tank from 0 to 10

since we took the sum of all the slices as $\Delta y \to 0$. Hence we obtained the integral.

$$\therefore W = \int_0^{10} \frac{62.4\pi}{100}(9)\, y^2 (y+10)\, dy$$

$$= \frac{62.4\pi(9)}{100}\int_0^{10} y^2(y+10)\,dy$$

$$= \frac{62.4\pi(9)}{100}\int_0^{10} y^3 + 10y^2\,dy$$

$$= \frac{62.4\pi(9)}{100}\left[\frac{y^4}{4} + \frac{10y^3}{3}\right]_0^{10}$$

$$= \frac{2808\pi}{500}\left[\frac{10^4}{4} + \frac{10(10)^3}{3}\right]$$

$$= \frac{2808\pi}{500}(10^4)\left[\frac{1}{4} + \frac{1}{3}\right]$$

$$= \frac{2808\pi}{500}(10^4)\left(\frac{7}{12}\right)$$

$= 32760\,\pi$ ft. pound.