Dawson College: **Calculus II**: 201-NYB-05-C1: Winter 2008

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Test 1

This test is graded out of 40 marks. No books, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Formula you might need:

$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin \frac{u}{a} + C$ $\qquad$ $\int \frac{du}{a^2+u^2} = \frac{1}{a} \arctan \frac{u}{a} + C$

$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a} \operatorname{arcsec} \frac{|u|}{a} + C$

Question 1. *(2 mark)*
Integrate the following indefinite integral:

$$\int (7\sin x + 3\cos x)\, dx = 7\int \sin x\, dx + 3\int \cos x\, dx$$

$$= -7\cos x + 3\sin x + C$$

Question 2. *(4 marks)*
Evaluate the following indefinite integral:

$$\int e^{9-x}\, dx \overset{1}{=} \int e^{u}\, dx$$

Let $u = 9 - x$ (1)

then $du = u'(x)\,dx$

$du = -dx$ (2)

$$\overset{2}{=} \int e^{u} \cdot -du$$

$$= -\int e^{u}\, du$$

$$= -e^{u} + C$$

$$\overset{1}{=} -e^{9-x} + C$$

Question 3. *(5 marks)*
Integrate the following indefinite integral:

$$\int \frac{1}{\theta^2} \cos \frac{1}{\theta}\, d\theta \overset{1}{=} \int \frac{1}{\theta^2} \cos u\, d\theta$$

Let $u = \frac{1}{\theta}$ (1)

then $du = u'(\theta)\, d\theta$

$du = \frac{-d\theta}{\theta^2}$

(2) $-du = \frac{d\theta}{\theta^2}$

$$\overset{2}{=} \int -\cos u\, du$$

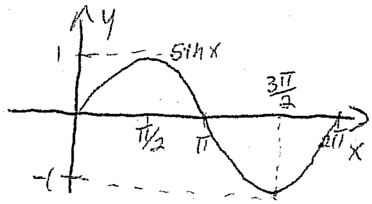
$$= -\int \cos u\, du$$

$$= -\sin u + C$$

Question 4. *(5 marks)*
Evaluate the following definite integral:

$$\int_0^{3\pi/2} \cos x \sin x \, dx$$

Let $u = \sin x$ (1)
then $du = \cos x \, dx$ (2)



$$\overset{(1)}{=} \int_0^{3\pi/2} (\cos x)\, u \, dx$$

$$\overset{(2)}{=} \int_{u(0)}^{u(\frac{3\pi}{2})} u \, du$$

$$= \int_0^{-1} u \, du$$

$$= \frac{u^2}{2} \Big|_0^{-1}$$

$$= \frac{(-1)^2}{2} - \frac{(0)^2}{2} = \frac{1}{2}$$

Question 5. *(5 marks)*
Integrate the following indefinite integral:

$$\int \frac{(\ln x)^2}{x} \, dx \overset{(1)}{=} \int \frac{(u)^2}{x} \, dx$$

Let $u = \ln x$ (1)
then $du = \frac{dx}{x}$ (2)

$$\overset{(2)}{=} \int u^2 \, du$$

$$= \frac{u^3}{3} + C$$

$$\overset{(1)}{=} \frac{(\ln x)^3}{3} + C$$

Question 6. *(3 marks)*
Given $\int_a^b f(x)\, dx = 5$, $\int_a^b g(x)\, dx = 3$ and $\int_b^c f(x)\, dx = 2$ evaluate the following definite integrals:

1.
$$\int_c^a f(x)\, dx = -\int_a^c f(x)\,dx = -\left[\int_a^b f(x)\,dx + \int_b^c f(x)\,dx\right] = -[5+2] = -7$$

2.
$$\int_b^a f(x) + g(x)\, dx = -\int_a^b f(x) + g(x)\,dx = -\left[\int_a^b f(x)\,dx + \int_a^b g(x)\,dx\right] = -[5+3] = -8$$

3.
$$\int_a^b g(x)\, dx = 3$$

Question 7. *(5 marks)*
Integrate the following indfinite integral *(hint: use inverse trigonometric funtion)*:

$$\int \frac{2}{3\sqrt{x}(4+x)}\,dx = \int \frac{2}{3\sqrt{x}\,(2^2+(\sqrt{x})^2)}\,dx$$

Let 1 $u=\sqrt{x}$ then

2 $du = \dfrac{dx}{2\sqrt{x}}$

$2du = \dfrac{dx}{\sqrt{x}}$

$$\overset{1}{=} \int \frac{2}{3\sqrt{x}\,(2^2+u^2)}\,dx$$

$$\overset{2}{=} \int \frac{2}{3\,(2^2+u^2)}\,2du$$

$$= \frac{4}{3}\int \frac{du}{(2^2+u^2)}$$

$$= \frac{4}{3}\left[\frac{1}{2}\arctan\frac{u}{2} + c\right]$$

$$= \frac{4}{6}\arctan\frac{\sqrt{x}}{2} + C$$

$$= \frac{2}{3}\arctan\frac{\sqrt{x}}{2} + C$$

Question 8. *(3 marks)*
Find the average value of the function $f(x) = 9x^2$ over the interval $[2,4]$:

$$\text{Avg value of } f = \frac{1}{b-a}\int_a^b f(x)\,dv$$

$$= \frac{1}{4-2}\int_2^4 9x^2\,dx$$

$$= \frac{1}{2}\left[3x^3\Big|_2^4\right]$$

$$= \frac{1}{2}\left[3(4)^3 - 3(2)^3\right]$$

$$= \frac{1}{2}[192-24]$$

$$= \frac{1}{2}[168]$$

$$= 84$$

Question 9. *(3 marks)*
Use the Second Fundamental Theorem of Calculus to find $F'(x)$.

$$F(x) = \int_0^{x^5} \cos y^5\,dy$$

Let 1 $u = x^5$

$$\frac{d}{dx}[F] = \frac{d}{du}\left[\int_0^u \cos y^5\,dy\right]\frac{d}{dx}[u]$$

$$= \cos u^5 \frac{d}{dx}[u]$$

by the second Fund. Thm. of Cal;

$$= \cos u^5\,(5x^4)$$

$$\overset{1}{=} 5(\cos x^{25})(x^4)$$

Question 10. *(5 marks)*
Evaluate the following definite integral:

$$\int_{-1}^{3} x(x^2+1)^4\,dx$$

Let ① $u = x^2 + 1$
then ③ $du = 2x\,dx$
$\frac{du}{2} = x\,dx$

$$\overset{①}{=} \int_{-1}^{3} x\,(u)^4\,dx$$

$$\overset{②}{=} \int_{u(-1)}^{u(3)} u^4 \frac{du}{2}$$

$$= \frac{1}{2}\int_{2}^{10} u^4\,du$$

$$= \frac{1}{2}\left[\frac{u^5}{5} \Big|_{2}^{10} \right]$$

$$= \frac{1}{10}\left[(10)^5 - (2)^5 \right]$$

$$= \frac{1}{10}\left[100\,000 - 32 \right]$$

$$= \frac{49\,984}{5}$$

Bonus Question. *(3 marks)*
Integrate the following indefinite intergral:

$$\int \frac{\ln\left(\frac{e}{x^2}\right)}{x}\,dx = \int \frac{\ln(e) - \ln x^2}{x}\,dx$$ rule of logs

$$= \int \frac{1 - 2\ln x}{x}\,dx$$ rule of logs

$$= \int \frac{1}{x}\,dx - 2\int \frac{\ln x}{x}\,dx$$

$$= \ln|x| - (\ln x)^2 + C$$