Dawson College: **Calculus II**: 201-NYB-05-C1: Winter 2008

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Test 2

This test is graded out of 43 marks. No books, notes, no graphing calculator or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(3 marks)*
Integrate the following indefinite integral:

$$\int t+\frac{1}{t}+\frac{1}{t^2}\,dt = \frac{t^2}{2} + \ln|t| - \frac{1}{t} + C$$

Question 2. *(5 marks)*
Integrate the following indefinite integral:

$$\int \sec^6 4x \tan 4x\,dx = \int \sec^5 4x \sec 4x \tan 4x\,dx$$

Let ① $u = \sec 4x$

$du = \sec 4x \tan 4x \, 4dx$

② $\frac{du}{4} = \sec 4x \tan 4x\,dx$

$$\overset{①}{=} \int u^5 \sec 4x \tan 4x\,dx$$

$$\overset{②}{=} \int u^5 \frac{du}{4}$$

$$= \frac{u^6}{24} + C$$

$$\overset{①}{=} \frac{(\sec 4x)^6}{24} + C$$

Question 3. *(3 marks)*
Integrate the following definite integral:

$$\int_1^4 \frac{x}{(1-x^2)^{\frac{2}{3}}}\,dx \overset{①}{=} \int_1^4 \frac{x}{u^{2/3}}\,dx$$

Let ① $u = 1-x^2$

$du = -2x\,dx$

② $\frac{du}{-2} = x\,dx$

$u(1) = 0$

$u(4) = 1-16$
$= 15$

$$\overset{②}{=} -\frac{1}{2}\int_0^{-15} \frac{du}{u^{2/3}}$$

$$= -\frac{1}{2}\left[3u^{1/3}\Big|_0^{-15}\right]$$

$$= -\frac{3}{2}(-15)^{1/3}$$

$$= \frac{3}{2}(15)^{1/3}$$

Question 4. *(3 marks)*
Integrate the following definite integral:

$$\int_1^4 \frac{u-2}{\sqrt{u}}\,du = \int_1^4 (u-2)u^{-1/2}\,du$$

$$= \int_1^4 u^{1/2} - 2u^{-1/2}\,du$$

$$= \left[\frac{2u^{3/2}}{3} - 4u^{\frac{1}{2}}\right]\Bigg|_1^4$$

$$= \frac{2(4)^{3/2}}{3} - 4(4)^{1/2} - \left[\frac{2}{3} - 4\right]$$

$$= \frac{2\cdot 8}{3} - 8 - \frac{2}{3} + 4$$

$$= \frac{2}{3}$$

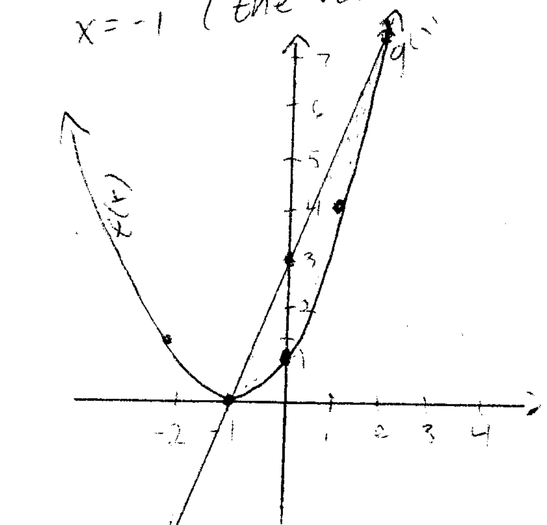
Question 5. *Sketch (2 marks) and area (2 marks)*
Sketch the graph of the algebraic function $f(x) = x^2 + 2x + 1$ and $g(x) = 3x + 3$ then find the area bounded by the two functions.

$f(x) = x^2 + 2x + 1$
$\quad = (x+1)^2$

$\therefore$ the vertex is on the x-axis at $(-1, 0)$

$f(x) = g(x)$
$x^2 + 2x + 1 = 3x + 3$
$x^2 - x - 2 = 0$
$(x-2)(x+1) = 0$

$\therefore$ the two function intersect at $x = 2$ and $x = -1$ (the vertex)



$$\text{Area} = \int_{-1}^{2} g(x) - f(x)\,dx$$

$$= \int_{-1}^{2} -x^2 + x + 2\,dx$$

$$= \left[\frac{-x^3}{3} + \frac{x^2}{2} + 2x\right]\Bigg|_{-1}^{2}$$

$$= -\frac{(2)^3}{3} + \frac{2^2}{2} + 2(2) - \left[-\frac{(-1)^3}{3} + \frac{(-1)^2}{2} + (2)(-\right.$$

$$= \frac{-8}{3} + \frac{4}{2} + 4 - \frac{1}{3} - \frac{1}{2} + 2$$

$$= \frac{9}{2}$$

$\therefore$ the area is $\frac{9}{2}$.

Question 6. *(5 marks)*
Integrate the following indefinite integral:

$$\int \frac{x^2}{x-1}\,dx$$

First we divide

$$\begin{array}{r|l} & x+1 \\ x-1 & x^2+0x+0 \\ & x^2-x \\ \hline & x \\ & x-1 \\ \hline & 1 \end{array}$$

$$\therefore \frac{x^2}{x-1} = x+1+\frac{1}{x-1}$$

$$= \int x+1+\frac{1}{x-1}\,dx$$

$$= \frac{x^2}{2} + x + \ln|x-1| + C$$

Question 7. *(5 marks)*
Integrate the following indefinite integral:

$$\int e^x \sin x\,dx$$

$$\int u\,dv = uv - \int v\,du$$

$u = e^x$ $\quad du = e^x dx$

$v = -\cos x$ $\quad dv = \sin x\,dx$

$$\int e^x \sin x\,dx = -e^x \cos x + \int e^x \cos x\,dx$$

Apply integration by parts again

$$\int e^x \sin x\,dx = -e^x \cos x + \left[uv - \int v\,du\right]$$

$u = e^x$ $\quad du = e^x dx$

$v = \sin x$ $\quad dv = \cos x\,dx$

$$\int e^x \sin x\,dx = -e^x \cos x + \left[e^x \sin x - \int e^x \sin x\,dx\right]$$

$$2\int e^x \sin x\,dx = e^x \sin x - e^x \cos x + C$$

$$\int e^x \sin x\,dx = \frac{1}{2} e^x [\sin x - \cos x] + C$$

Question 8. *(5 marks)*
Integrate the following indefinite integral:

$$\int \sin^2 x \cos^2 x \, dx$$

$$= \int \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right) dx$$

$$= \frac{1}{4}\int (1-\cos^2 2x)\, dx$$

$$= \frac{1}{4}\int \sin^2 2x \, dx$$

$$= \frac{1}{4}\int \frac{1-\cos 4x}{2}\, dx$$

$$= \frac{1}{8}\int 1-\cos 4x \, dx$$

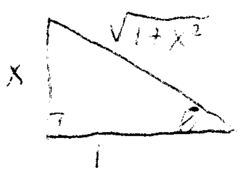
$$= \frac{1}{8}\left[x - \frac{\sin 4x}{4}\right] + C$$

Question 9. *(5 marks)*
Integrate the following indefinite integral:

$$\int \frac{9x^3}{\sqrt{1+x^2}}\, dx$$

Let ① $x = \tan\theta$

② $dx = \sec^2\theta \, d\theta$



Let ③ $u = \sec\theta$

④ $du = \sec\theta\tan\theta \, d\theta$

$\sec\theta = \dfrac{h}{a}$

$= \sqrt{1+x^2}$

$$\overset{①}{=} 9\int \frac{\tan^3\theta}{\sqrt{1+\tan^2\theta}}\, dx$$

$$\overset{②}{=} 9\int \frac{\tan^3\theta \sec^2\theta}{\sqrt{\sec^2\theta}}\, d\theta$$

$$= 9\int \frac{\tan^3\theta \sec^{\cancel{2}}\theta}{\cancel{\sec^2\theta}}\, d\theta$$

$$= 9\int \tan^2\theta \sec\theta \tan\theta \, d\theta$$

$$= 9\int (\sec^2\theta - 1)\sec\theta\tan\theta \, d\theta$$

$$\overset{③④}{=} 9\int (u^2-1)\, du$$

$$= 9\left[\frac{u^3}{3} - u\right] + C$$

$$= 9\left[\frac{\sec^3\theta}{3} - \sec\theta\right] + C$$

$$= 3\left(\sqrt{1+x^2}\right)^3 - 9\sqrt{1+x^2} + C$$

Question 10. *(5 marks)*
Integrate the following indefinite integral:

$$\int \frac{2x^2+5x+1}{x^3+2x^2+x}\,dx$$

Lets use partial fractions $\frac{2x^2+5x+1}{x^3+2x^2+x} = \frac{2x^2+5x+1}{x(x+1)^2}$

$$\frac{2x^2+5x+1}{x(x+1)^2} = \frac{A}{x} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2}$$

$$2x^2+5x+1 = A(x+1)^2 + Bx(x+1) + Cx$$

Let $x=0 \Rightarrow$
$2(0)^2+5(0)+1 = A(0+1)^2$
$1 = A$

Let $x=-1 \Rightarrow$
$2(-1)^2+5(-1)+1 = C(-1)$
$2 = C$

Let $x=1$
$2(1)^2+5(1)+1 = 1(1+1)^2 + B(1)(1+1) + 2(1)$
$8 = 4 + 2B + 2$
$1 = B$

$$\therefore \int \frac{1}{x} + \frac{1}{x+1} + \frac{2}{(x+1)^2}\,dx$$

$$= \ln|x| + \ln|x+1| - 2\frac{1}{(x+1)} + C$$

Bonus Question. *(3 marks)*
Integrate the following indefinite intergral:

$$\int \frac{1}{2t\cos^2 t + t - t\cos 2t}\,dt = \int \frac{1}{t(2\cos^2 t + 1 - \cos 2t)}\,dt$$

$$= \int \frac{1}{t\left(2\left(\frac{1+\cos 2t}{2}\right)+1-\cos 2t\right)}\,dt$$

$$= \int \frac{1}{2t}\,dt$$

$$= \frac{1}{2}\ln|t| + C$$